UNIVERSITÉ DE MONTRÉAL

CENTRE DE RECHERCHE SUR LES TRANSPORTS

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

L-1.2 :
A PROGRAM FOR BOX-COX TRANSFORMATIONS IN REGRESSION MODELS WITH HETEROSKEDASTIC AND AUTOREGRESSIVE RESIDUALS

by

Tran Cong Liem
Marcel G. Dagenais
Marc J.I. Gaudry

This work was supported at various stages by the Transport Canada, by the F.C.A.R. program and the M.E.S.S.T. of Quebec, and by the S.S.H.R.C. and the N.S.E.R.C. of Canada.

Cette étude a été publiée grâce à une subvention du fonds F.C.A.R. pour l'aide et le soutien à la recherche.

Université de Montréal

Centre de recherche sur les transports - Publication #510
Département de Sciences économiques - Cahier #8714

Janvier 1987

Dépôt légal - 2e trimestre 1987

CONTENTS

Page

ABSTRACT. iii

I. THE LIKELIHOOD FUNCTION 1

II. THE CONCENTRATED LOG-LIKELIHOOD FUNCTION. 2

III. PROGRAM DESCRIPTION 5

IV. INPUT FORMATS 7

V. INPUT PARAMETERS. 8

VI. DESCRIPTION OF OUTPUT 14

1) Scaling of Variables. 14

2) Expected Values of the Dependent Variable 14
a) Transformed Variable (y**) 14
b) Original Variable (y). 16

3) Measures of the Coefficient of Determination. 18
a) Pseudo-(E)-R^2. 18
b) Pseudo-(L)-R^2. 18

4) Elasticity Coefficients and Derivatives of the Dependent Variable with Respect to the Independent Variables. 19
a) Elasticity Coefficient in Terms of the Sample Value of of y_t. 19
b) Elasticity Coefficient in Terms of $E(y_t)$ 20
c) Derivatives of the Dependent Variable with Respect to the Independent Variables. 21

5) Autocorrelation Analysis for Residuals. 21

6) Plot of Residuals Against One Independent Variable. 23

7) Constraints on the Autocorrelation Parameters 23

8) Likelihood Ratio Test 24

9) Asymptotic Covariance Matrix for the Parameters 24

10) Correlation Matrices and Detection of Multicollinearity . . . 27

11) Student-t-Statistics. 29

Page

12) Forecasting 30
a) Maximum Likelihood Forecast. 30
b) Simulation Forecast. 36
c) Special Case without Box-Cox Transformation on the Dependent Variable 38

VII. SPECIAL OPTIONS 39

1) Box-Cox Transformation on Quasi-Dummies 40

2) Retrieval of the Estimation Results for Tabulation. 40

3) Tabulation : Program TABLEX 41

VIII. SAMPLE DECK STRUCTURE AND PRINT-OUT 45

PRINT-OUT 51

BIBLIOGRAPHY. 81

ABSTRACT

L-1.2 is a program designed to deal with the specification of the functional form in the generalized single-equation regression model when the functional form of heteroskedasticity of the residuals, which may also be autocorrelated, is fully analysed. Fixed or estimated parameters in the Box-Cox transformations can be associated with the dependent and groups of independent variables, which may be simultaneously modified by a chosen autocorrelation structure in which the autoregressive parameters are estimated, and by a selected functional form of heteroskedasticity in which the different parameters can be either fixed or estimated. The asymptotic covariance matrix of all parameter estimates of the system is computed. A number of useful outputs, including elaborate statistics associated with forecasts and plots of the likelihood functions, can also be obtained.

- - - - - - - - - - -

RÉSUMÉ

Le programme L-1.2 est destiné à l'étude de la forme fonctionnelle du modèle de régression multiple lorsque la forme de l'hétéroscédasticité des erreurs, qui peuvent aussi être autocorrelées, doit aussi être analysée en profondeur. Le programme permet de fixer ou d'estimer les paramètres dans les transformations Box-Cox appliquées à la variable dépendante et aux groupes de variables indépendantes, d'estimer conjointement les paramètres d'une structure d'autocorrélation choisie et finalement de fixer ou d'estimer les différents paramètres associés à la forme de l'hétéroscédasticité des erreurs. La matrice de covariance asymptotique des estimations de tous ces paramètres est calculée. La procédure permet aussi de faire un certain nombre de calculs utiles, dont des prévisions et des graphes de la fonction de vraisemblance.

I. The Likelihood Function

Consider the following model specified by Gaudry and Dagenais (1978) and summarized in Gaudry and Dagenais (1979) :

$$y_t^{(\lambda_y)} = \sum_k \beta_k X_{kt}^{(\lambda_{xk})} + u_t \quad , \qquad (t=1,\ldots,n) \qquad (1)$$

$$u_t = \sqrt{f(X_t)}\, v_t \quad , \qquad (2)$$

$$v_t = \sum_{\ell=1}^{r} \rho_\ell\, v_{t-\ell} + w_t \quad , \qquad (3)$$

where $E(v_t^2) = \phi^2$, the r-th autoregressive process is assumed to satisfy standard stationarity assumptions and the w_t are normally and independently distributed with zero mean and covariance matrix $\sigma_w^2 I$.

The functional form for heteroskedasticity is assumed to be :

$$f(X_t) = \begin{cases} \exp\left(\sum_m \delta_m X_{mt}^{(\lambda_{em})}\right) & \text{if } \lambda_u = 0 \\ \left[\lambda_u \left(\delta_o + \sum_m \delta_m X_{mt}^{(\lambda_{em})}\right) + 1\right]^{1/\lambda_u} > 0 & \text{if } \lambda_u \neq 0 \quad , \end{cases} \qquad (4)$$

where $y_t^{(\lambda_y)}$, $X_{kt}^{(\lambda_{xk})}$ and $X_{mt}^{(\lambda_{em})}$ are respectively the Box-Cox transforms of y_t, X_{kt} and X_{mt}; if Z_t denotes y_t, X_{kt} or X_{mt}, the Box-Cox transformation of $Z_t > 0$ is defined for the parameter λ as :

$$Z_t^{(\lambda)} = \begin{cases} \ln Z_t & \text{if } \lambda = 0 \quad , \\ (Z_t^\lambda - 1)/\lambda & \text{if } \lambda \neq 0 \quad . \end{cases} \qquad (5)$$

Note that $f(X_t)$ is in the form of an inverse Box-Cox transformation, and the set of variables X_m may be partly or completely different from the X_k's group. We will not allow the case $\lambda_u \neq 0$ in this program due to insufficient experience with it so far.

The likelihood function associated with this model, when the first r observations are dropped in order to reduce the considerable computational burden which keeping them would have involved, can be expressed as :

$$L = \prod_{t=1+r}^{n} \frac{1}{\sigma_w\sqrt{2\pi}} \frac{y_t^{\lambda_y - 1}}{\sqrt{f(X_t)}} \exp\left\{-\frac{1}{2\sigma_w^2}\left(\frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r} \rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}}\right)^2\right\} , \quad (6)$$

where $u_t = y_t^{(\lambda_y)} - \sum_k \beta_k X_{kt}^{(\lambda_{xk})}$.

The corresponding log-likelihood function is :

$$\text{Log } L = -\frac{n-r}{2} \ln (2\pi\sigma_w^2) + (\lambda_y - 1) \sum_{t=1+r}^{n} \ln y_t - \frac{1}{2} \sum_{t=1+r}^{n} \ln f(X_t)$$

$$- \frac{1}{2\sigma_w^2} \sum_{t=1+r}^{n} \left(\frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r} \rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}}\right)^2 . \quad (7)$$

II. The Concentrated Log-likelihood Function

In view of problems involving large numbers of parameters, specially in the fixed part of model (1) — the β_k's —, the log-likelihood function (7) must be concentrated first on σ_w^2 and then on the β_k's, in order to reduce the number of parameters directly involved in the maximization of (7) : the remaining parameters to be estimated are ρ, λ_x, λ_y, δ and λ_e, where the underscripts ℓ, k, m are dropped for ρ, λ_x, δ and λ_e to indicate the corresponding column vectors.

Taking the first derivative of (7) with respect to σ_w^2, yields :

$$\frac{\partial \text{ Log } L}{\partial \sigma_w^2} = -\frac{n-r}{2}\frac{1}{\sigma_w^2} + \frac{1}{2\sigma_w^4}\sum_{t=1+r}^{n}\left(\frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r}\rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}}\right)^2$$

$$= \frac{1}{2\sigma_w^2}\left[-(n-r) + \frac{1}{\sigma_w^2}\sum_{t=1+r}^{n}\left(\frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r}\rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}}\right)^2\right] . \qquad (8)$$

Setting (8) to zero, we solve for σ_w^2 :

$$\hat{\sigma}_w^2 = \sum_{t=1+r}^{n}\left(\frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r}\rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}}\right)^2/(n-r) . \qquad (9)$$

Replacing σ_w^2 by its value $\hat{\sigma}_w^2$ in formula (7), we obtain the log-likelihood function concentrated on σ_w^2 :

$$\text{Log } L^* = -\frac{n-r}{2}[1+\ln(2\pi)] - \frac{n-r}{2}\ln\hat{\sigma}_w^2 + (\lambda_y - 1)\sum_{t=1+r}^{n}\ln y_t - \frac{1}{2}\sum_{t=1+r}^{n}\ln f(X_t). \qquad (10)$$

To concentrate the function (10) on the β_k's, we must use matrix notation for the expression in (9), which can be rewritten as :

$$\hat{\sigma}_w^2 = \frac{1}{n-r} w'w \quad , \qquad (11)$$

where w is a column vector containing the last n-r residuals w_t, (t=1+r,...,n) :

$$w_t = \frac{u_t}{\sqrt{f(X_t)}} - \sum_{\ell=1}^{r}\rho_\ell \frac{u_{t-\ell}}{\sqrt{f(X_{t-\ell})}} = y_t^{**} - \sum_k X_{kt}^{**}\beta_k \qquad (12)$$

and

$$\begin{cases} y_t^{**} = y_t^{*} - \sum_{\ell} \rho_\ell \, y_{t-\ell}^{*} & , \\ X_{kt}^{**} = X_{kt}^{*} - \sum_{\ell} \rho_\ell \, X_{k,t-\ell}^{*} & , \\ y_t^{*} = y_t^{(\lambda_y)} / \sqrt{f(X_t)} & , \\ X_{kt}^{*} = X_{kt}^{(\lambda_{xk})} / \sqrt{f(X_t)} & . \end{cases} \tag{13}$$

In terms of the β_k's, $\hat{\sigma}_w^2$ can be expressed as :

$$\hat{\sigma}_w^2 = \frac{1}{n-r} (y^{**} - X^{**} \beta)' (y^{**} - X^{**} \beta) \quad ,$$

where y^{**} is a column vector containing the last n-r observations on y_t^{**}: similarly, X^{**} is a $(n-r) \times q$ matrix and β is a q-element column vector.

The first derivative of (10) with respect to vector β is :

$$\frac{\partial \, \text{Log } L^*}{\partial \, \beta} = - \frac{n-r}{2} \frac{1}{\hat{\sigma}_w^2} \frac{\partial \, \hat{\sigma}_w^2}{\partial \, \beta} = - \frac{1}{\hat{\sigma}_w^2} \left[\frac{\partial (y^{**} - X^{**}\beta)'}{\partial \, \beta} \right] (y^{**} - X^{**}\beta)$$

$$= \frac{1}{\hat{\sigma}_w^2} X^{**\prime} (y^{**} - X^{**}\beta) = \frac{1}{\hat{\sigma}_w^2} (X^{**\prime} y^{**} - X^{**\prime} X^{**}\beta) \quad . \tag{15}$$

Setting (15) to zero, we obtain for β :

$$\hat{\beta} = (X^{**\prime} X^{**})^{-1} X^{**\prime} y^{**} \quad . \tag{16}$$

Replacing β by its value $\hat{\beta}$ in the expression (14), we obtain :

$$\hat{\sigma}_w^2 = \frac{1}{n-r} (y^{**} - X^{**} \hat{\beta})' (y^{**} - X^{**} \hat{\beta}) \quad . \tag{17}$$

Finally, the log-likelihood function concentrated on σ_w^2 and β can be written as :

$$\text{Log } L^{**} = \frac{n-r}{2}[1+\ln(2\pi)] - \frac{n-r}{2}\ln\hat{\sigma}_w^2 + (\lambda_y - 1)\sum_{t=1+r}^{n}\ln y_t - \frac{1}{2}\sum_{t=1+r}^{n}\ln f(X_t). \quad (18)$$

III. Program Description

To maximize the function (18) with respect to ρ, λ_x, λ_y, δ and λ_e, we use a minimization algorithm due to Fletcher and Powell (1963).

The main features of the program can be summarized as follows :

1. for the fixed part of model (1), the independent variables X_k's can be subdivided into a few groups, each of which contains the X_k's affected by a common λ_{xk} so as to reduce the number of λ_{xk}'s, which would be otherwise as great as the number of X_k's. For some of the groups, the λ_{xk}'s can be fixed and for the others, the λ_{xk}'s are to be estimated. Moreover, there is one particular group which is left untouched by the Box-Cox transformation, namely the group containing the constant term, the dummy variables and the ordinary variables which are not strictly positive for all observations;

2. the constraints of type $\lambda_y = \lambda_{x(i)}$ is allowed where i represents the i-th group of X_k's and when both parameters are estimated;

3. in the functional form for heteroskedasticity (4), the δ_m's and λ_{em}'s can be either fixed or estimated;

4. the initial values for every estimated λ_{xk} and λ_{em} can be set equal to unity. The same is true for estimated λ_y;

5. the initial value for every ρ_ℓ and for every δ_m can be set equal to zero in the case of autocorrelation and heteroskedasticity; in the absence of autocorrelation and heteroskedasticity, the ρ_ℓ's and the δ_m's need not be specified;

6. the main reason for choosing the initial values set in 4. and 5. is to start with a linear model without considering the problem of autocorrelation and heteroskedasticity;

7. numerically, the transformation $X_t^{(\lambda)}$ defined in (5) is switched to the form $\ln Z_t$ when $|\lambda|<10^{-4}$;

8. to avoid numerical problems, all the estimated λ's are constrained over a region such that $|\lambda|< 20$ (see SUBROUTINE TRANSF) and for the case of heteroskedasticity, when $\lambda_u = 0$. the argument for the exponential defined in (4) is bounded such that $\sum_m \delta_m X_{mt}^{(\lambda_{em})} < 650$ (see SUBROUTINE FCN), which is approximately the range of the argument for the exponential function on a CDC-CYBER-835/855.

IV. Input Formats

Almost all input is in free format, except for TITREY, Y, TITREX and X which are in user specified format.

card type	input contents
1	_$CARD1_NOBS=..,NC=..,NF=..,NFP=..,NOBEL=..,NM=..,NSTEP=..,EPSI=..,WGHT=..$ (80 columns; first column and column following $CARD1 must be blanks. Input values must be specified for every parameter.)
2	_$CARD2_NB=..,NRO=..,NSL=..,NGF=..,MAXF=..,NGV=..,MAXV=..,NLY=..,NDM=.., NLE=.., ND=..$ (Same remarks as for CARD1)
3	_$CARD3_NCOR=..,NRES=..,NPW=..,NGUVW=..,NPDQL=..,NCOV=..,NSTORE=..,NGRID=.., NPASS=..,NLSQ=..$ (Same remarks as for CARD1)
4	80 columns for comments (first column must be blank)
(5)	(L(I),I=1,NRO),(RO(I),I=1,NRO) (if NRO ≠ 0)
(6)	(VLXF(I),I=1,NGF),(NXF(I),I=1,NGF) (if NGF ≠ 0)
(7)	(VLXV(I),I=1,NGV),(NXV(I),I=1,NGV) (if NGV ≠ 0)
(8)	VLY (if NLY=0 or 1) or NGXY (if NLY=2),ULY (if NLY ≠ 1)
(9)	(D(I),I=1,ND),(VLE(I),I=1,ND),(NUMX(I),I=1,ND) (if NDM ≠ -1)
(10)	(NUMG(I),I=1,NGUVW) (if NGUVW ≠ 0)
(11)	NQM,NYEAR (if NPW ≠ 0)
(12)	NDIFF,NDLSEA,LNPER,NLAG,NLAGP (if NPDQL ≠ 0)
(13)	(IFMT(I),I=1,10) (80 columns)
(14)	TITREY,(Y(I),I=1,NOBS)
(15)	TITREX(J),(X(I,J),I=1,NOBS) (Repeat for J=1,NB+NM)
(16)	TITY TITX NAMP(1),NUMP(1),DPAR(1) NTYPE(1),NGROUP(1) (This card is only required when NAMP(1)=BE) NAMP(2),NUMP(2),DPAR(2) NTYPE(2),NGROUP(2) (This card is only required when NAMP(2)=BE) (if NSTORE=2; Repeat this card sequence for NGRID grids)

NOTE : The card type number in parentheses indicates that the input card is optional and conditional on the control parameter value specified in cards no. 2 and 3.

V. Input Parameters

- Card type no. 1 : 9 parameters

NOBS : total number of observations in the sample

NC : index number of the first observation in the estimation period

NF : index number of the last observation in the estimation period (NC<NF<NOBS)

NFP : index number of the last observation in the forecast period; there are NFP-NF periods of forecast (NF<NFP<NOBS). If NFP=NF, no forecast is required.

NOBEL : number of observations used in the last portion of the estimation period, for the computation of the elasticity coefficients at the means of the dependent and independent variables

NM : number of X_m's not belonging to the X_k's group for the heteroskedasticity case.

NSTEP : maximum number of steps for the maximization of the function. (Usually, NSTEP can be set at least equal to the square of the number of parameters in the concentrated log-likelihood function)

EPSI : convergence parameter for the function (10^{-7}<EPSI<10^{-4})

WGHT : common weighting factor for the changes in the parameters during the maximization of the concentrated log-likelihood function. (Usually, WGHT=1., but other values such as 0.1, 0.01, 0.001 can be chosen when there is a problem of convergence)

- Card type no. 2 : 11 parameters

NB : number of estimated β_k's, excluding the constant β_o

NRO : number of estimated ρ_ℓ's

NSL : number of X_k's not affected by the Box-Cox transformation, excluding the constant (0<NSL<50)

NGF : number of X_k's groups with fixed λ_{xk}'s

MAXF : maximum number of variables X_k's among the NGF groups (if NGF=0, MAXF must be set equal to zero)

NGV : number of X_k's groups with estimated λ_{xk}'s

MAXV : maximum number of variables X_k's among the NGV groups (if NGV=0, MAXV must be set equal to zero)

NLY : control parameter for λ_y:

NLY=-1 : no λ_y on the dependent variable

NLY=0 : λ_y is fixed during the maximization

NLY=1 : λ_y is estimated

NLY=2 : λ_y is estimated and set equal to one of the λ_{xk}'s of the NGV groups

NDM : control parameter for the δ_m's :

NDM=-1 : heteroskedasticity is not considered (NLE must be set equal to zero)

NDM=0 : the δ_m's are fixed during the maximization

NDM=1 : the δ_m's are estimated

NLE : control parameter for the λ_{em}'s

NLE=0 : the λ_{em}'s are fixed during the maximization

NLE=1 : the λ_{em}'s are estimated

ND : number of variables X_m's appearing in $f(X_t)$. (If NDM=-1, NLE and ND must be set equal to zero; up to ten variables can be used)

- Card type no. 3 : 10 parameters

NCOR =

0 : no print of correlation matrices and variance decomposition proportions

1 : print correlation matrix and variance decomposition proportions in terms of X** and y** at the maximum point of the log-likelihood function

2 : print correlation matrices and variance decomposition proportions in terms of X and y, and X** and y** respectively at the starting point and the maximum point of the log-likelihood function

NRES =

0 : no print and no plot of residuals w_t

1 : print and plot of residuals at the maximum point of the log-likelihood function

2 : print and plot of residuals at the starting and maximum points of the log-likelihood function

NPW =
- 0 : no list of residuals w_t by quarters or months
- 1 : list of residuals by quarters (one output line per year)
- 2 : list of residuals by months (one output line per year)

NGUVW =
- 0 : no plot of independent variables against residuals u_t, v_t and w_t
- value : number of independent variables to be plotted against u_t, v_t and w_t (up to ten variables can be used)

NPDQL =
- 0 : no print of autocorrelation and partial autocorrelation functions
- 1 : print at the maximum point of the log-likelihood function
- 2 : print at the starting and maximum points of the log-likelihood function

NCOV =
- 0 : no print of covariance matrix of the parameters at the maximum point of the log-likelihood function
- 1 : print covariance matrix

NSTORE =
- 0 : no storage of function and parameters values on TAPE3, at the maximum point of the log-likelihood function
- 1 : storage of function and parameters values on TAPE3 (unformatted WRITE), to be used in later runs with NSTORE=2
- 2 : read from TAPE3 the function and parameters values (unformatted READ) when NGRID≠0

NGRID =
- 0 : no grid when NSTORE=0 and 1
- value : number of grids for two estimated parameters around the maximum point of the log-likelihood function

NPASS = number of the run (maximum two digits)

NLSQ =
- 0 : normal run, when there is at least one parameter to be estimated other than the β coefficients, i.e. the p's, λ_y, λ_x's, δ's or λ_e's. The estimated results are stored on TAPE1 for later use in program TABLEX (see Chapter VII, Section 2).
- 1 : special run, when there are no other parameters to be estimated than the β coefficients, i.e. when there is no autocorrelation and all the λ_y, λ_x's, δ's and λ_e's are either fixed or not considered. The β coefficients are just estimated by an ordinary least squares regression. The estimated results are stored on TAPE1 for later use in program TABLEX.

- Card type no. 4

80 columns for comments (first column must be blank)

- Card type no.(5): necessary only if NRO≠0.

L : autocorrelation orders associated with the ρ_ℓ's (in ascending order)

RO : initial values for the ρ_ℓ's

- Card type no.(6) : necessary only if NGF≠0.

VLXF : fixed values for the λ_{xk}'s in the NGF groups

NXF : number of X_k's per group, for the NGF groups

- Card type no.(7) : necessary only if NGV≠0.

VLXV : initial values for the λ_{xk}'s in the NGV groups

NXV : number of X_k's per group, for the NGV groups

- Card type no.(8) : necessary only if NLY≠-1.

VLY : fixed value (if NLY=0) or initial value (if NLY=1) for λ_y

NGXY : group number for the X_k's with $\lambda_{xk} = \lambda_y$ (if NLY=2)

ULY : upper limit ν (before scaling), for the dependent variable y in the computation of $E(y_t)$ when $\lambda_y < 0$, in (29). If the user does not want to specify this limit, he simply sets ULY=0 and a default value of ν will be computed from (30).

- Card type no.(9) : necessary only if NDM≠-1.

D	:	fixed (if NDM=0) or initial (if NDM=1) values for δ_m's
VLE	:	fixed (if NLE=0) or initial (if NLE#1) values for λ_{em}'s
NUMX	:	variable numbers for the X_m's entering in $f(X_t)$

- Card type no.(10) : necessary only if NGUW≠0.

NUMG	:	variable numbers of the independent variables to be plotted against the residuals

- Card type no.(11) : necessary only if NPW≠0.

NQM	:	quarter (1,...,4) or month (1,...,12) associated with the first observation in the sample
NYEAR	:	year (four digits) associated with the first observation in the sample

- Card type no.(12) : necessary only if NPDQL≠0.

NDIFF	:	degree of consecutive differencing (parameter d as defined in Chapter VI, Section 5)
NDLSEA	:	degree of seasonal differencing (parameter D)
LNPER	:	period of seasonal differencing (parameter s)
NLAG	:	maximum lag for autocorrelation function (parameter K)
NLAGP	:	maximum lag for partial autocorrelation function (parameter L)

- Card type nos. 13, 14 and 15

IFMT	:	input format within parentheses for TITREY, Y and TITREX, X
TITREY	:	variable name for y
Y	:	observations on y_t(t=1,NOBS); when the forecast option is chosen (NFP>NF), make sure that observations on y_t(t=NF+1,NOBS) are always present : if they are not available, a zero value must be input for each of them
TITREX	:	variable name for X_k or X_m
X	:	observations on X_{kt} or X_{mt}(t=1,NOBS)

- Card type no.(16) : necessary only if NSTORE=2.

TITY : parameter label for the VERTICAL axis in the grid : FORMAT (A10)

TITX : parameter label for the horizontal axis in the grid : FORMAT (A10)

NAMP(I)
NUMP(I)
DPAR(I) : are respectively the coded name, the index number and the step size of the parameter (I=1 for the parameter on the vertical axis and I=2 for the parameter on the horizontal axis) : FORMAT (A2,I2,F10.1)

The following coded names (two characters) are adopted for the parameters :

parameter	β_k	λ_{xk}	λ_y	δ_m	λ_{em}	ρ_ℓ
NAMP(I)	BE	LX	LY	DE	DE	RO

To assign two different parameters to the axes, the order of priority for the parameters on the vertical-horizontal axes must be respected : β_k must precede λ_{xk} must precede λ_y and so on. For example, NAMP(1)=BE and NAMP(2)=DE constitute a valid choice; but the converse is illegal (NAMP(1)=DE and NAMP(2)=BE).

The index number of the parameter, NUMP, is the location number of the element with respect to each vector β, λ_x, λ_y, δ, λ_e or ρ.

If a grid is desired for λ_y with any other parameter, in the case of NLY=2, one has to specify the λ_{xk} which is set equal to λ_y, instead of λ_y itself, on the NAMP, NUMP, DPAR card. Note that the grid only applies for two estimated parameters.

The step size DPAR can be chosen in the range 0.10 to 0.01 times the standard-error of the parameter. The grid is computed for nine points in each direction, and centered at the maximum point of the log-likelihood function.

NTYPE(I)
NGROUP(I) : are respectively the type and the group of the λ_{xk} corresponding to the β_k which is chosen as NAMP(I)=BE : FORMAT (A2,I2). This card is not required when NAMP(I)≠BE.

The type has the following form :

NTYPE(I) = { NL : No Lambda
FL : Fixed Lambda
EL : Estimated Lambda

The group of the λ_{xk} is chosen among the NGF or NGV groups. When NTYPE(I)=NL, NGROUP(I) must be set equal to zero, since there is no lambda associated with this group.

NOTE : The program creates automatically the constant which occupies the last position within the set of X_k's, i.e. the (NB+1)-th location. The variable numbers for the X_k's are given by the positions corresponding to the order of appearance of the X_k's in the set. If there are some X_m's which do not belong to the X_k's group (NM≠0), they must be added at the end of this group, and their corresponding positions (hence, their variable numbers), must be counted as NB+1+i (i=1,NM).

VI. Description of Output

1) Scaling of Variables :

To avoid numerical problems, a change of scale for y_t, X_{kt} and X_{mt} is performed by using powers of 10 as the scaling factors :

$$Z_t' = Z_t / 10^{[\log_{10} \bar{Z}]} , \qquad (19)$$

where Z_t represents y_t, X_{kt} or X_{mt}; $\bar{Z}$ is the mean of Z, computed only from the subset of positive observations, and $[\log_{10} \bar{Z}]$ is the greatest integer contained in $\log_{10} \bar{Z}$.

2) Expected Values of the Dependent Variable :

a) Transformed Variable (y**)

Since the Box-Cox transformation applies only to stricly positive variables, and in particular for the dependent variable y_t, it will be convenient to assume for the moment that $\varepsilon < y_t < \nu$, where ε is an arbitrary small positive number and ν is the upper limit of y_t.

Following (12) and (13), we can write :

$$y_t^{**} = \sum_k X_{kt}^{**} \beta_k + w_t = \frac{y_t^{\lambda_y} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_\ell \rho_\ell \, y_{t-\ell}^{*} \qquad (20)$$

The above assumption for y_t implies that

$$\frac{\varepsilon^{\lambda_y} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \, y^*_{t-\ell} < y^{**}_t < \frac{\nu^{\lambda_y} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \, y^*_{t-\ell} \,, \quad (\lambda_y > 0) \tag{21.a}$$

and

$$\frac{1/\varepsilon^{|\lambda_y|} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \, y^*_{t-\ell} < y^{**}_t < \frac{1/\nu^{|\lambda_y|} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \, y^*_{t-\ell} \,, \quad (\lambda_y < 0) \tag{21.b}$$

Relaxing the assumption ($\varepsilon \to 0$ and $\nu \to \infty$), we obtain :

$$\begin{cases} \tilde{y}^{**}_t < y^{**}_t < \infty \,, & (\lambda_y > 0) \qquad (22.a) \\ -\infty < y^{**}_t < \tilde{y}^{**}_t \,, & (\lambda_y < 0) \qquad (22.b) \end{cases}$$

where $\tilde{y}^{**}_t = - \dfrac{1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \, y^*_{t-\ell}$.

The expected value of y^{**}_t can then be expressed as :

$$E(y^{**}_t) = \begin{cases} \tilde{y}^{**}_t \int_{-\infty}^{\tilde{w}^{**}_t} N(w)\, dw + \int_{\tilde{w}^{**}_t}^{\infty} (\sum_k X^{**}_{kt} \beta_k + w)\, N(w)\, dw \,, & (\lambda_y > 0) \quad (23.a) \\ \int_{-\infty}^{\tilde{w}^{**}_t} (\sum_k X^{**}_{kt} \beta_k + w)\, N(w)\, dw + \tilde{y}^{**}_t \int_{\tilde{w}^{**}_t}^{\infty} N(w)\, dw \,, & (\lambda_y < 0) \quad (23.b) \end{cases}$$

where

$$\tilde{w}^{**}_t = \tilde{y}^{**}_t - \sum_k X^{**}_{kt} \beta_k \quad \text{and} \quad N(w) = (2\pi \sigma_w^2)^{-1/2} \exp(-w^2/2\sigma_w^2) \,.$$

When $\lambda_y = 0$, it is readily seen that $-\infty < y^{**}_t < \infty$ and we have :

$$E(y^{**}_t) = \int_{-\infty}^{\infty} (\sum_k X^{**}_{kt} \beta_k + w)\, N(w)\, dw \,. \tag{24}$$

Note that $E(y_t^{**})$ can also be computed as :

$$E(y_t^{**}) = \hat{y}_t^{**} + E(w_t) \qquad (25)$$

where

$$E(w_t) = \begin{cases} \tilde{w}_t^{**} \int_{-\infty}^{\tilde{w}_t^{**}} N(w)\, dw + \int_{\tilde{w}_t^{**}}^{\infty} w\, N(w)\, dw & , \ (\lambda_y > 0) \\ 0 & , \ (\lambda_y = 0) \\ \int_{-\infty}^{\tilde{w}_t^{**}} w\, N(w)\, dw + \tilde{w}_t^{**} \int_{\tilde{w}_t^{**}}^{\infty} N(w)\, dw & , \ (\lambda_y < 0) \end{cases}$$

b) Original Variable (y)

Reassuming that $\varepsilon < y_t < \nu$ and extending Tobin's (1958) formula to a doubly censored dependent variable [Dagenais, Gaudry and Liem (1980)], the expected value of y_t can be written as follows :

$$E(y_t) = \varepsilon \int_{-\infty}^{w_t(\varepsilon)} N(w)\, dw + \int_{w_t(\varepsilon)}^{w_t(\nu)} y_t(w)\, N(w)\, dw + \nu \int_{w_t(\nu)}^{\infty} N(w)\, dw \qquad (26)$$

where

$$w_t(\xi) = \begin{cases} \dfrac{\xi_t^{\lambda_y} - 1}{\lambda_y \sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell\, y_{t-\ell}^{*} - \sum_{k} X_{kt}^{**}\, \beta_k & , \ (\lambda_y \neq 0) \\ \dfrac{\ell n(\xi)}{\sqrt{f(X_t)}} - \sum_{\ell} \rho_\ell \dfrac{\ell n(y_{y-\ell})}{\sqrt{f(X_{t-\ell})}} - \sum_{k} X_{kt}^{**}\, \beta_k & , \ (\lambda_y = 0) \end{cases}$$

and

$$y_t(w) \begin{cases} \left[1 + \lambda_y \sqrt{f(X_t)} \left(\sum_{\ell} \rho_\ell \, y^*_{t-\ell} + \sum_k X^{**}_{kt} \, \beta_k + w\right)\right]^{1/\lambda_y} & , \ (\lambda_y \neq 0) \\ \exp\left[\sqrt{f(X_t)} \left(\sum_{\ell} \rho_\ell \frac{\ell n(y_{t-\ell})}{\sqrt{f(X_{t-\ell})}} + \sum_k X^{**}_{kt} \, \beta_k + w\right)\right] & , \ (\lambda_y \neq 0) \ . \end{cases}$$

When $\lambda_y > 0$, as $\varepsilon \to 0$ and $\nu \to \infty$, the first and last terms on the RHS of (26) vanish and the expression reduces to :

$$E(y_t) = \int_{\tilde{w}^{**}_t}^{\infty} y_t(w) \, N(w) \, dw \ . \qquad (27)$$

For the case $\lambda_y = 0$, the expression (26) can be simplified in the same way, as $\varepsilon \to 0$ and $\nu \to \infty$:

$$E(y_t) = \int_{-\infty}^{\infty} y_t(w) \, N(w) \, dw \ . \qquad (28)$$

When $\lambda_y < 0$, as $\varepsilon \to 0$, $w_t(\varepsilon) \to -\infty$ and the first term on the RHS of (26) disappears :

$$E(y_t) = \int_{-\infty}^{w_t(\nu)} y_t(w) \, N(w) \, dw + \nu \int_{w_t(\nu)}^{\infty} N(w) \, dw \ . \qquad (29)$$

If we let now ν tend to infinity, then $w_t(\nu) \to \tilde{w}^{**}_t$ and $E(y_t)$ does not have any finite value. In that case, the experimenter is asked to supply a reasonable value of ν for which $E(y_t)$ can be computed. If the value of ν is not specified, the program will generate a default value which is set as follows :

$$\nu = \min \left[\max (y_t) + 10 \, \sigma_y, \ (- 0.8 \, \sigma_w \, \lambda_y)^{1/\lambda_y}\right] \ , \qquad (30)$$

where σ_y is the sample standard error of y_t.

The mean probability of y to be at the limit (ε for $\lambda > 0$ or ν for λ 0) is used as a sample measure of the degree of approximation of $E(y_t)$ in (27) and (29) :

$$\overline{Pr}(y < \varepsilon) = \frac{1}{n-r} \sum_{t=1+r}^{n} \int_{-\infty}^{w_t(\varepsilon)} N(w)\, dw) \quad , \quad (\lambda_y > 0)$$

and (31)

$$\overline{Pr}(y > \nu) = \frac{1}{n-r} \sum_{t=1+r}^{n} \int_{w_t(\nu)}^{\infty} N(w)\, dw) \quad , \quad (\lambda_y < 0) \quad .$$

At the initial and final steps of the log-likelihood function maximization, the expected value of y_t, $E(y_t)$, is always plotted against the residual w_t in standardized form (w_t/σ_w). A point in the plot is indicated by its frequency : an asterisk (*) represents a point with a frequency greater than nine.

3) Measures of the Coefficient of Determination

Two measures of R^2 are computed in the program.

a) Pseudo-(E)-R^2 :

$$R^2_E = 1 - \sum_{t=1+r}^{n} (y_t - E(y_t))^2 / \sum_{t=1+r}^{n} (y_t - \bar{y})^2 \quad , \qquad (32)$$

where $E(y_t)$ is computed from (27), (28), and (29), depending on the value of λ_y. Note that when $\lambda_y < 0$, this measure can become negative if the default value of ν or the one specified by the user happens to be so high that $E(y_t)$ can take on very big values.

b) Pseudo-(L)-R^2 :

$$R^2_L = 1 - \Lambda^{2/n} \qquad (33)$$

where $N = n-r$ and $\Lambda = \max_{\Pi \varepsilon \omega} L(\Pi) / \max_{\Pi \varepsilon \Omega} L(\Pi)$ is the ratio of two maximum likelihood functions, one corresponding to the region ω in the parameter space Ω,

specified under the null hypothesis H : $\tilde{\beta} = 0$ for the linear case without autocorrelation and heteroskedasticity ($\tilde{\beta}$ is the regression coefficient vector containing the elements of β, except the coefficient for the constant term), and the other associated with the more or less restricted parameter space Ω for $\Pi = (\beta', \rho', \lambda_x', \lambda_y, \delta', \lambda_e', \sigma_w^2)'$.

An explicit form of $\Lambda^{2/N}$ in terms of y_t and y_t^{**} can be obtained :

$$\Lambda^{2/N} = \frac{\sum_{t=1+r}^{n} (y_t^{**} - \hat{y}_t^{**})^2}{\sum_{t=1+r}^{n} (y_t - \bar{y})^2} \left\{ \sum_{t=1+r}^{n} [y_t^{\lambda_y - 1} / \sqrt{f(X_t)}] \right\}^{-2/N} \qquad (34)$$

It is readily seen that R_L^2 is identical to R_E^2 in the classical linear regression case without autocorrelation and heteroskedasticity.

4) <u>Elasticity Coefficients and Derivatives of the Dependent Variable with Respect to the Independent Variables</u>

Two types of elasticity coefficients are computed in the program.

a) <u>Elasticity Coefficient in Terms of the Sample Value of y_t</u>

$$\eta_{jt}^{(s)} = \frac{\partial y_t}{\partial X_{jt}} \frac{X_{jt}}{y_t} = \frac{1}{y_t^{\lambda_y}} (X_{jt}^{\lambda_{xj}} \beta_j + G_{jt}) \quad , \qquad (35)$$

where

$$G_{jt} = \begin{cases} 0 & , \text{ if } f(X_t) \text{ is not a function of } X_{jt} ; \\ \frac{1}{2} X_{mt}^{\lambda_{em}} \delta_m (y_t^{(\lambda_y)} - \sum_k X_{kt}^{(\lambda_{xk})} \beta_k) & , \text{ if } f(X_t) \text{ contains one } X_{mt} = X_{jt} . \end{cases}$$

The superscript (s) indicates that the elasticity is derived with respect to the sample value of y_t. The program computes the elasticity coefficient at the means of two different sets of observations on the dependent and independent variables :

i) means of observations for the whole estimation period, namely observations from period t+r to n;

ii) means of observations for the last portion of the estimation period, namely for the last NOBEL periods.

b) Elasticity Coefficient in Terms of $E(y_t)$

$$\eta_{jt}^{(e)} = \frac{\partial E(y_t)}{\partial X_{jt}} \frac{X_{jt}}{E(y_t)}$$

$$\eta_{jt}^{(e)} = \frac{1}{E(y_t)} \int_{R(W)} [y_t(w)]^{1-\lambda_y} [X_{jt}^{\lambda_{xj}} \beta_j + H_{jt}(w)] N(w)\, dw \quad , \qquad (36)$$

where

$$H_{jt}(w) = \begin{cases} 0 & , \text{ if } f(X_t) \text{ is not a function of } X_{jt} ; \\ \frac{1}{2} X_{mt}^{\lambda_{em}} \delta_m \{[y_t(w)]^{(\lambda_y)} - \sum_k X_{kt}^{(\lambda_{xk})} \beta_k\} & , \text{ if } f(X_t) \text{ contains one } X_{mt} = X_{jt} ; \end{cases}$$

and R(w) is the domain of integration $[\tilde{w}_t^{**}, \infty]$, $[-\infty, \infty]$ or $[-\infty, w_t(v)]$ for w when $\lambda_y > 0$, $\lambda_y = 0$ or $\lambda_y < 0$ respectively.

The superscript (e) indicates that the elasticity is derived with respect to the expected value of y_t, $E(y_t)$, which is given in (27), (28) and (29) depending on the value of λ_y. Unlike the elasticity $\eta_{jt}^{(s)}$, the elasticity $\eta_{jt}^{(e)}$ is also a function of the lagged values of the dependent and independent variables : when the elasticity coefficient is computed at the means, the unlagged values are replaced by their means as in the case of $\eta_{jt}^{(s)}$, but the lagged values must be replaced by the means of observations lagged for ℓ periods, for each set of observations.

When $\lambda_y = 0$ and $f(X_t)$ is not a function of X_{jt}, $\eta_{jt}^{(s)}$ and $\eta_{jt}^{(e)}$ are identical and equal to $X_{jt}^{\lambda_{xj}} \beta_j$; furthermore, if $\lambda_{xj} = 0$, we obtain the well known result when both y_t and X_{jt} are in logarithmic form, namely $\eta_{jt}^{(s)} = \eta_{jt}^{(e)} = \beta_j$.

c) Derivatives of the Dependent Variable with Respect to the Independent Variables

The derivatives $\partial y_t / \partial X_{jt}$ and $\partial E(y_t) / \partial X_{jt}$ which are implicitly computed in (35) and (36) at the sample means of the variables are also printed in terms of their original units : the derivatives with respect to the constant term are meaningless, so they are not computed but set equal to zero in the program. Note that when the model is linear ($\lambda_y = \lambda_{xk} = 1$ for every k), these derivatives correspond exactly to the regression coefficients β associated with the variabes in their original units.

5) Autocorrelation Analysis for Residuals

At the initial and final stages of the maximization of Log L** given in (18), autocorrelation and partial autocorrelation functions are evaluated for the residual term w_t defined in (12).

Following the notation of Box-Jenkins (1976), a differencing operation is first applied to the series w_t :

$$\tilde{w}_t = (1 - B)^d \, (1 - b^s)^D \, w_t \quad , \quad (t=1+r,\ldots,n) \qquad (37)$$

where B is the backward shift operator defined by $Bw_t = w_{t-1}$, hence $B^m w_t = w_{t-m}$, d is the degree of consecutive differencing, D is the degree of seasonal differencing and s is the period of seasonal differencing.

The following values are printed out :

a) Autocorrelation Function :

$$r_k = c_k/c_0 \quad , \quad k=0,1,\ldots,K \quad , \tag{38}$$

where c_k is the autocovariance function of lag k :

$$c_k = \sum_{t=1+r}^{n-k} (\tilde{w}_t - \bar{\tilde{w}})\,(\tilde{w}_{t+k} - \bar{\tilde{w}})/(n-r) \quad , \tag{39}$$

$\bar{\tilde{w}}$ is the mean of $\tilde{w}_t$ and K is the maximum lag of autocovariance and autocorrelation functions. Note that c_0 is the variance of $\tilde{w}_t$.

b) Partial Autocorrelation Function :

$$\hat{\phi}_{\ell\ell} = \begin{cases} r_1 \quad , & \text{for } \ell=1 \quad , \\ \dfrac{r_\ell - \sum_{j=1}^{\ell-1} \hat{\phi}_{\ell-1,j}\, r_{\ell-j}}{1 - \sum_{j=1}^{\ell-1} \hat{\phi}_{\ell-1,j}\, r_j} \quad , & \ell=2,3,\ldots,L \quad , \end{cases} \tag{40}$$

where L is the maximum lag of partial autocorrelation function, and

$$\hat{\phi}_{\ell j} = \hat{\phi}_{\ell-1,j} - \hat{\phi}_{\ell\ell}\, \hat{\phi}_{\ell-1,\ell-j} \quad , \quad j=1,2,\ldots,\ell-1 \quad .$$

c) Estimated standard errors for the autocorrelations and partial autocorrelations are printed next to each row of output and apply to all entries in that row.

Both consecutive and seasonal differencing may be applied more than once or together to produce higher order differences. In the program, the maximum values of d, D, s, K, and L are respectively 3, 3, 31, 96, 36. The allowed ranges for d, D, s, K and L are :

$0 < d < 3$, $0 < D < 3$, $0 < s < 31$, $d + SD < n-r$, and $K > L$,

K and L must be multiples of 12 (K=12,24,...,96 and L=12,24,36).

6) Plot of Residuals Against One Independent Variable

At the initial and final steps of the maximization process, the estimated residuals of u_t, v_t and w_t are plotted around their means, against the observations ordered by increasing values of one selected independent variable, to provide a graphical analysis of the functional form of heteroskedasticity related to this variable.

The residuals are plotted to scale on the horizontal axis, whereas on the vertical axis, the independent variable is not, due to the length of the series : one line skipping represents an observation, independently of the value of the observation.

7) Constraints on the Autocorrelation Parameters

During the maximization procedure for Log L**, a constraint on each parameter $\rho_\ell (\ell=1,\ldots,r)$ is introduced implicitly by using the Fisher's z-transformation :

$$z_\ell = \frac{1}{2} \ln \frac{1 + \rho_\ell}{1 - \rho_\ell} , \quad -1 < \rho_\ell < 1 \quad \text{and} \quad -\infty < z_\ell < +\infty \quad . \qquad (41)$$

In other words, we have changed a maximization problem with constraints into one without constraints : the output parameter values between iterations are thus in terms of z, λ_x, λ_y, δ and λ_e. Note that the inverse transformation of (41) is $\rho_\ell = \tanh z_\ell$, and $\frac{\partial \text{ Log } L}{\partial z_\ell} = \frac{\partial \text{ Log } L}{\partial \rho_\ell} \frac{\partial \rho_\ell}{\partial z_\ell}$ where $\frac{\partial \text{ Log } L}{\partial \rho_\ell}$ is given in (43) and $\frac{\partial \rho_\ell}{\partial z_\ell} = \text{sech}^2 z_\ell = 1 - \tanh^2 z_\ell = 1 - \rho_\ell^2$.

8) Likelihood Ratio Test

At the end of the maximization process, a likelihood ratio test is performed by computing the following statistic :

$$-2 \, (\text{Log } L_0^{**} - \text{Log } L_1^{**}) \quad , \tag{42}$$

which is distributes asymptotically as χ^2, with a number of degrees of freedom equal to the difference in the numbers of parameters involved in the two models, where L_0^{**} is the maximum of the likelihood function associated with the initial model (at the beginning of the iterations) and L_1^{**} is the maximum of the likelihood function associated with the final model (at the end of the iterations).

9) Asymptotic Covariance Matrix for the Parameters

To compute the asymptotic covariance matrix for the parameters β, ρ, λ_x, λ_y, δ, λ_e and σ_w^2 at the maximum point of the likelihood function, we use the fundamental statistical relation that the asymptotic covariance matrix of a maximum likelihood estimator is equal to the inverse of the covariance matrix of the gradient of the likelihood function (Kendall and Stuart, p. 55, vol II (1967); for an application of the theorem in a nonlinear structural model, see Berndt, Hall et al. (1974)).

The gradient of the log-likelihood function (7) can be written as :

$$\frac{\partial \text{ Log } L}{\partial \Pi_i} = \sum_{t=1+r}^{n} D_{t,i} \quad , \tag{43}$$

where Π_i represents a typical element of β, ρ, λ_x, λ_y, δ, λ_e and σ_w^2; $D_{t,i}$ is defined as :

$$D_{t,i} = \begin{cases} \dfrac{\partial(\lambda_y - 1)}{\partial \Pi_i} \ln y_t - \dfrac{1}{2} \dfrac{\partial \ln f(X_t)}{\partial \Pi_i} - \dfrac{w_t}{\sigma_w^2} \dfrac{\partial w_t}{\partial \Pi_i} & \text{if } \Pi_i \neq \sigma_w^2 \\ \left(\dfrac{w_t^2}{\sigma_w^2} - 1 \right) / \, 2\sigma_w^2 & \text{if } \Pi_i = \sigma_w^2 \end{cases} \tag{44}$$

At the maximum of the log-likelihood function, the asymptotic covariance matrix of the parameters is estimated by :

$$COV = \left[\sum_{t=1+r}^{n} D_t \, D_t' \right]^{-1} \tag{45}$$

where D_t is a column vector with $D_{t,i}$ as a typical element.

Note that the covariance matrix is output in a lower triangular form.

The different derivatives involved in the expression for $D_{t,i}$, when $\Pi_i \neq \sigma_w^2$, are given below.

$$\frac{\partial(\lambda_y - 1)}{\partial \Pi_i} = \begin{cases} 0 & \text{if } \Pi_i \neq \lambda_y \ , \\ 1 & \text{if } \Pi_i = \lambda_y \ . \end{cases} \tag{46}$$

Π_i	$\partial w_t / \partial \Pi_i$
β_k	$- X^{**}_{kt}$
ρ_ℓ	$- (y^{*}_{t-\ell} - \sum_k X^{*}_{k,t-\ell} \, \beta_k)$
λ_{xk}	$- \beta_k \left[\frac{1}{\sqrt{f(X_t)}} \frac{\partial X_{kt}^{(\lambda_{xk})}}{\partial \lambda_{xk}} - \sum_\ell \frac{\rho_\ell}{\sqrt{f(x_{t-\ell})}} \frac{\partial X_{k,t-\ell}^{(\lambda_{xk})}}{\partial \lambda_{xk}} \right]$
λ_y	$\frac{1}{\sqrt{f(X_t)}} \frac{\partial y_t^{(\lambda_y)}}{\partial \lambda_y} - \sum_\ell \frac{\rho_\ell}{\sqrt{f(x_{t-\ell})}} \frac{\partial X_{t-\ell}^{(\lambda_y)}}{\partial \lambda_y}$

(cont'd) :

Π_i	$\partial w_t / \partial \Pi_i$
δ_m	$-\frac{1}{2}\left[(y^*_t - \sum_k X^*_{kt}\,\beta_k)\,X^{(\lambda_{em})}_{mt} - \sum_\ell \rho_\ell\,(y^*_{t-\ell} - \sum_k X^*_{k,t-\ell}\,\beta_k)\,X^{(\lambda_{em})}_{m,t-\ell}\right]$
λ_{em}	$-\frac{1}{2}\left[\delta_m (y^*_t - \sum_k X^*_{kt}\beta_k)\,\frac{\partial X^{(\lambda_{em})}_{mt}}{\partial \lambda_{em}} - \sum_\ell \rho_\ell\,(y^*_{t-\ell} - \sum_k X^*_{k,t-\ell}\,\beta_k)\,\frac{\partial X^{(\lambda_{em})}_{m,t-\ell}}{\partial \lambda_{em}}\right]$

Π_i	$\partial \ln f(X_t)/\partial \Pi_i$
β_k	0
ρ_ℓ	0
λ_{xk}	0
λ_y	0
δ_m	$X^{(\lambda_{em})}_{mt}$
λ_{em}	$\delta_m \frac{\partial X^{(\lambda_{em})}_{mt}}{\partial \lambda_{em}}$

To compute the different derivatives $\partial y_t^{(\lambda_y)}/\partial\lambda_y$, $\partial X_{kt}^{(\lambda_{xk})}/\partial \lambda_{xk}$ and $\partial X_{mt}^{(\lambda_{em})}/\partial \lambda_{em}$, the following formulas are used :

$$\frac{\partial Z_t^{(\lambda)}}{\partial \lambda} = \begin{cases} \frac{1}{2} (\ln Z_t)^2 & , \text{ if } \lambda = 0 \\ \frac{1}{\lambda} (Z_t^{\lambda} \ln Z_t - Z_t^{(\lambda)}) & , \text{ if } \lambda \neq 0 \end{cases} \tag{47}$$

The derivatives for the case $\lambda = 0$ are actually computed for very small values of λ, say $|\lambda| < 10^{-4}$.

10) Correlation Matrices and Detection of Multicollinearity

A correlation matrix for the independent variables (excluding the constant) and the dependent variable in terms of the original variables (X and y) is always given before the maximization procedure begins. Another correlation matrix in terms of the transformed variables (X** and y**) is finally computed at the maximum of the log-likelihood function.

The matrices are stored and output in a lower triangular form, where the last row represents the pairwise correlations between each of the independent variables and the dependent variable.

To detect the presence of multiple linear dependencies among the original independent variables (X), the spectral decomposition of X'X is used (Judge et al., 1985). This method is similar to the singular value decomposition of the matrix X (Belsley, Kuh and Welsch, 1980). The analysis is also performed for the transformed variables (X**) at the maximum point of the log-likelihood function.

The spectral decomposition of X'X is defined as :

$$X'X = \sum_{i=1}^{K} \lambda_i \, p_i p_i' \tag{48}$$

where p_i is the i-th eigenvector (Kx1) associated with the i-th eigenvalue λ_i of X'X and the columns of X are scaled to unit length but not centered around their sample means, because centering obscures any linear dependency that involves the constant term.

Belsley et al. use a set of condition indexes which is a generalization of the concept of the condition number of a matrix to detect the presence of near dependencies among the columns of X :

i) Condition number of X : $\kappa(X) = (\lambda_{max}/\lambda_{min})^{1/2}$, which is a measure of the sensitivity of β to changes in X'X or X'y;

ii) Condition indexes : $\eta_i = (\lambda_{max}/\lambda_i)^{1/2}$, (i=1,...,K).

To determine which variables are involved in each near dependency, a decomposition of the variance of β is performed :

$$\text{var}(\beta_k) = \sigma^2 \sum_{j=1}^{K} (p_{kj}^2/\lambda_j) \text{ , } (k=1,\ldots,K) \quad . \tag{49}$$

The proportion of var (β_k) associated with any λ_i is then computed :

$$\Pi_{ik} = (p_{ki}^2/\lambda_i)/(\sum_{j=1}^{K} p_{kj}^2/\lambda_j) \quad . \tag{50}$$

These results can be summarized in a table of variance-decomposition proportions, where the elements in each column are reordered according to the increasing values of the λ_i's :

Eigenvalue	Condition index	Variance-decomposition proportions var(β_1)	var(β_2)	. . .	var(β_K)
$\lambda_1(\lambda_{min})$	$\eta_1 = \kappa(X)$	Π_{11}	Π_{12}	. . .	Π_{1K}
λ_2	η_2	Π_{21}	Π_{22}	. . .	Π_{2K}
.	.	.	.		.
.	.	.	.		.
.	.	.	.		.
$\lambda_K(\lambda_{max})$	$\eta_K = 1$	Π_{K1}	Π_{K2}	. . .	Π_{KK}

The sum of the proportions in each column is equal to one. The following rules of thumb can be used to detect the presence of near dependencies :

i) High values of the condition indexes ($\eta > 30$) signal the existence of near dependencies, while high associated Π's in excess of 0.5 indicate which variable is involved in the collinear relations.

ii) When a given variable is involved in several relations, its proportions can be individually small across the high η's. In this case, the sum of these proportions which is in excess of 0.5 also diagnose variable involvement.

11) Student-t-Statistics

There are two tables where the asymptotic t-statistics are reported :

i) The first table given at the initial step includes only the t-tests for the regression coefficients $\hat{\beta}$, conditional upon the initial or fixed values of ρ, λ_x, λ_y. δ amd λ_e : in other words, the table simply reports the standard results of the regression of y** on X**. Those tests are invariant to changes of measurement units in y and X.

ii) The second table computed at the maximum likelihood estimates of β, ρ, λ_x, λ_y, δ, λ_e and σ_w^2 includes two rows of t-tests for each parameter :
- the first row corresponds to the unconditional t-test computed under the null hypothesis that the parameter is equal to zero, i.e. using the standard errors derived from (45). Except for β and δ, these tests are invariant to changes of measurement units in y and X.
- the second row contains two types of t-tests which are both invariant: for the Box-Cox parameters λ_x, λ_y and λ_e, the unconditional t-test is computed under the null hypothesis that the parameters is equal to one, and for the remaining parameters β, ρ, δ and σ_w^2, the t-test is computed under the null hypothesis that the parameter is equal to

zero, conditionally upon the estimates of λ_x, λ_y and λ_e (Dagenais, Gaudry and Liem, 1985), i.e. using the inverse of the submatrix derived from $[\sum_{t=1+r}^{n} D_t D_t']$ by deleting the rows and columns associated with λ_x, λ_y and λ_e to obtain the conditional covariance matrix of β, ρ, δ and σ_w^2.

When the program fails to converge to a maximum point, the covariance matrix of the parameters cannot be estimated; then the standard errors will be set equal to unity so that the final table can be printed to give information about the gradient : in that case, all the Student-t statistics shown are not correct.

12) Forecasting

a) Maximum Likelihood Forecast

To obtain the predicted values of the dependent variable over the p periods from n+1 to n+p, the following approach is adopted : for the first period of forecast, the predicted value of y_{n+1} is given by the first-order condition $\partial \text{ Log } L_{n+1}/\partial y_{n+1} = 0$, since maximizing $\sum_{t=1+r}^{n+1} \text{Log } L_t$ with respect to y_{n+1} is equivalent to maximizing the log-likelihood function in the (n+1)-th period only.

The first derivative of the log-likelihood function associated with the (n+1)-th period, with respect to y_{n+1}, can be written as :

$$\frac{\partial \text{ Log } L_{n+1}}{\partial y_{n+1}} = \frac{1}{y_{n+1}} \left[\lambda_y - 1 - \frac{y_{n+1}^{\lambda_y}}{\sigma_w^2 \sqrt{f(X_{n+1})}} \left(\frac{u_{n+1}}{\sqrt{f(X_{n+1})}} - \sum_{\ell} \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}} \right) \right] \quad (51)$$

where $u_{n+1} = y_{n+1}^{(\lambda_y)} - \sum_k X_{k,n+1}^{(\lambda_{xk})} \beta_k$.

Setting this derivative equal to zero and solving for $y_{n+1}^{\lambda_y}$, we obtain an equation of the second degree in $y_{n+1}^{\lambda_y}$:

$$y_{n+1}^{2\lambda_y} + B\, y_{n+1}^{\lambda_y} + C = 0 \quad , \quad \text{if } \lambda_y \neq 0 \tag{52}$$

where

$$B = -\left[1 + \lambda_y \sqrt{f(X_{n+1})}\left(\sum_k \beta_k \frac{X_{k,n+1}^{(\lambda_{xk})}}{\sqrt{f(X_{n+1})}} + \sum_\ell \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}}\right)\right]$$

and

$$C = -\lambda_y\,(\lambda_y - 1)\,\sigma_w^2\, f(X_{n+1}) \quad .$$

Since the predicted value of y_{n+1} must be real and positive, the values of $y_{n+1}^{\lambda_y} = (-B \pm \sqrt{B^2 - 4C})/2$ are subject to the same restrictions, namely $B^2 - 4C > 0$ and $y_{n+1}^{\lambda_y} > 0$. If the two roots of $y_{n+1}^{\lambda_y}$ are both real, the greater root will be always chosen as shown in the next section. Finally, the obtained solution for y_{n+1} must also satisfy the second order condition that $\partial^2 \text{ Log } L_{n+1} / \partial\, y_{n+1}^2$ be negative.

When $\lambda_y = 0$, the derivative in (51) can be rewritten as :

$$\frac{\partial \text{ Log } L_{n+1}}{\partial\, y_{n+1}} = \frac{1}{y_{n+1}}\left[-1 - \frac{1}{\sigma_w^2 \sqrt{f(X_{n+1})}}\left(\frac{u_{n+1}}{\sqrt{f(X_{n+1})}} - \sum_\ell \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}}\right)\right], \tag{53}$$

where $u_{n+1} = \ln y_{n+1} - \sum_k X_{k,n+1}^{(\lambda_{xk})} \beta_k$.

Setting this derivative equal to zero, a unique solution for y_{n+1} is found :

$$\hat{y}_{n+1} = \exp\left[\sum_k X_{k,n+1}^{(\lambda_{xk})} \beta_k + \sqrt{f(X_{n+1})} \sum_\ell \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}} - \sigma_w^2\, f(X_{n+1})\right] . \tag{54}$$

Clearly, the approach can be easily extended to the next periods, since in each single period of forecast, only the log-likelihood function associated with that period is needed for the maximization, given the sample observed values of the dependent variable in the estimation period and its predicted values in the preceding forecast periods. Thus, for the p-th period of forecast, the predicted value of y_{n+p} is also given by the first order condition $\partial \text{ Log } L_{n+p}/\partial\, y_{n+p} = 0$, subject to the second order condition $\partial^2 \text{ Log } L_{n+p}/\partial\, y^2_{n+p} < 0$. The formulas are analogous to (51), (52) (53) and (54) with n+1 replaced by n+p.

The variance of the forecast error for the first period of forecast is computed from the covariance matrix of the parameters Π and y_{n+1}, which is equal to minus the inverse matrix of the second derivatives of the log-likelihood function $\text{Log } L_{(1)} = \sum_{t=1+r}^{n+1} \text{Log } L_t$ with respect to Π and y_{n+1}, evaluated at the maximum point of the function :

$$\text{COV}_{(1)} = -\begin{bmatrix} \partial^2 \text{ Log } L_{(1)}/\partial\, \Pi\, \partial\, \Pi' & \partial^2 \text{ Log } L_{(1)}/\partial\, \Pi\, \partial\, y_{n+1} \\ \partial^2 \text{ Log } L_{(1)}/\partial\, y_{n+1}\, \partial\, \Pi' & \partial^2 \text{ Log } L_{(1)}/\partial\, y^2_{n+1} \end{bmatrix}^{-1} , \quad (55)$$

where Π is a column vector of the parameters β, ρ, λ_x, λ_y, δ, λ_e and σ^2_w, and the underscript in parentheses (1) indicates that the first period of forecast is concerned.

By the rule of partitioned matrix inversion, the variance of the forecast error for y_{n+1} can be obtained :

$$\sigma^2_{n+1} = -\left[\frac{\partial^2 \text{ Log } L_{(1)}}{\partial\, y^2_{n+1}}\right]^{-1} + \left[\frac{\partial^2 \text{ Log } L_{(1)}}{\partial\, y^2_{n+1}}\right]^{-1} \left[\frac{\partial^2 \text{ Log } L_{(1)}}{\partial\, y_{n+1}\, \partial\, \Pi'}\right] \text{COV}_{(o)} \left[\frac{\partial^2 \text{ Log } L_{(1)}}{\partial\, \Pi\, \partial\, y_{n+1}}\right] \left[\frac{\partial^2 \text{ Log } L_{(1)}}{\partial\, y^2_{n+1}}\right]^{-1} , \quad (56)$$

where $\text{COV}_{(o)}$ is the covariance matrix of Π already estimated in (45).

Proceeding in a recursive manner, the variance of the forecast error for the p-th period can be written as :

$$\sigma^2_{n+p} = -\left[\frac{\partial^2 \text{Log } L_{(p)}}{\partial\, y^2_{n+p}}\right]^{-1} + \left[\frac{\partial^2 \text{Log } L_{(p)}}{\partial\, y^2_{n+p}}\right]^{-1} \left[\frac{\partial^2 \text{Log } L_{(p)}}{\partial\, y_{n+p}\, \partial\, \Pi'_{(p-1)}}\right] \text{COV}_{(p-1)} \left[\frac{\partial^2 \text{Log } L_{(p)}}{\partial\, \Pi_{(p-1)}\, \partial\, y_{n+p}}\right]\left[\frac{\partial^2 \text{Log } L_{(p)}}{\partial\, y^2_{n+p}}\right]^{-1} , \tag{57}$$

where $\text{COV}_{(p-1)}$ is the covariance matrix of $\Pi_{(p-1)} = (\Pi', y_{n+1}, \ldots, y_{n+p-1})'$ and $\text{Log } L_{(p)} = \sum_{t=1+r}^{n+p} \text{Log } L_t$. Note that the second derivatives

$\partial^2 \text{Log } L_{(p)} / \partial\, \Pi_{(p-1)}\, \partial\, y_{n+p}$ and $\partial^2 \text{Log } L_{(p)} / \partial\, y^2_{n+p}$ are respectively equal to $\partial^2 \text{Log } L_{(n+p)} / \partial\, \Pi_{(p-1)}\, \partial_{n+p}$ and $\partial^2 \text{Log } L_{(n+p)} / \partial\, y^2_{n+p}$ since y_{n+p} appears only in $\text{Log } L_{n+p}$.

The second derivatives of $\text{Log } L_{n+p}$ with respect to $\Pi_{(p)} = (\Pi', y_{n+1}, \ldots, y_{n+p})'$ and y_{n+p} are in the form $y^{\lambda_y - 1}_{n+p} / [\sigma^2_w \sqrt{f(X_{n+p})}]$ times an expression which is given below for each case :

parameter	y_{n+p}
β_k	$\dfrac{X^{(\lambda_{xk})}_{k,n+p}}{\sqrt{f(X_{n+p})}} - \sum_{\ell} \rho_\ell \dfrac{X^{(\lambda_{xk})}_{k,n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}}$
ρ_ℓ	$\dfrac{X^{(\lambda_y)}_{n+p-\ell} - \sum_{k} \beta_k X^{(\lambda_{xk})}_{k,n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}}$

(cont'd) :

parameter	y_{n+p}
λ_{xk}	$\beta_k \left[\frac{1}{\sqrt{f(X_{n+p})}} \frac{\partial X_{k,n+p}^{(\lambda_{xk})}}{\partial \lambda_{xk}} - \sum_{\ell} \frac{\rho_\ell}{\sqrt{f(X_{n+p-\ell})}} \frac{\partial X_{k,n+p-\ell}^{(\lambda_{xk})}}{\partial \lambda_{xk}} \right]$
λ_y	$\frac{\sigma_w^2 \sqrt{f(X_{n+p})}}{y_{n+p}^{\lambda_y}} - w_{n+p} \ln (y_{n+p}) - \left[\frac{1}{\sqrt{f(X_{n+p})}} \frac{\partial y_{n+p}^{(\lambda_y)}}{\partial \lambda_y} - \sum_{\ell} \frac{\rho_\ell}{\sqrt{f(X_{n+p-\ell})}} \frac{\partial y_{n+p-\ell}^{(\lambda_y)}}{\partial \lambda_y} \right]$
δ_m	$\frac{1}{2}\left[\left(\frac{2\, u_{n+p}}{\sqrt{f(X_{n+p})}} - \sum_{\ell} \rho_\ell \frac{u_{n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}} \right) X_{m,n+p}^{(\lambda_{em})} - \sum_{\ell} \rho_\ell \frac{u_{n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}} \frac{\partial y_{n+p-\ell}^{(\lambda_y)}}{\partial \lambda_y} \right]$
λ_{em}	$\frac{\delta_m}{2}\left[\left(\frac{2\, u_{n+p}}{\sqrt{f(X_{n+p})}} - \sum_{\ell} \rho_\ell \frac{u_{n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}} \right) \frac{\partial X_{m,n+p}^{(\lambda_{em})}}{\partial \lambda_{em}} - \sum_{\ell} \rho_\ell \frac{u_{n+p-\ell}}{\sqrt{f(X_{n+p-\ell})}} \frac{\partial X_{m,n+p-\ell}^{(\lambda_{em})}}{\partial \lambda_{y_{em}}} \right]$
σ_w^2	w_{n+p}/σ_w^2
y_{n+p-s}	$\frac{\rho_s}{\sqrt{f(X_{n+p-s})}} y_{n+p-s}^{\lambda_y - 1}$, where s=1,...,min (p-1,r) and p > 2
y_{n+p}	$-(2\, y_{n+p}^{\lambda_y} + B)/(y_{n+p} \sqrt{f(X_{n+p})})$

The different derivatives $\partial\, y_{n+p}^{(\lambda_y)} / \partial\, \lambda_y$, $\partial\, X_{k,n+p}^{(\lambda_{xk})} / \partial\, \lambda_{xk}$ and $\partial\, X_{m,n+p}^{(\lambda_{em})} / \partial\, \lambda_{em}$ can be computed, using the formulas shown in (47).

In evaluating the second derivatives of Log L_{n+p} at the maximum point of the log likelihood function Log $L_{(p)}$, the predicted values of $y_{n+1}, \ldots, y_{n+p}$ obtained by the approach above, are used just as when the predicted value of y_{n+p} is computed, the predicted values of $y_{n+1}, \ldots, y_{n+p-1}$ are also used.

Finally, the following recursive formula can be used to construct the covariance matrix of $\Pi_{(i)}$, at the i-th forecast period :

$$COV_{(i)} = \begin{bmatrix} COV_{(i-1)} & C_{(i)} \\ C'_{(i)} & \sigma^2_{n+i} \end{bmatrix} , \qquad (58)$$

where σ^2_{n+i} is the variance of the forecast error for y_{n+i}, and $C_{(i)}$ is the column vector of covariances between $\Pi_{(i-1)}$ and y_{n+i} :

$$C_{(i)} = - COV_{(i-1)} \left[\frac{\partial^2 \text{ Log } L_{(i)}}{\partial\, \Pi_{(i-1)}\, \partial\, y_{n+i}} \right] \left[\frac{\partial^2 \text{ Log } L_{(i)}}{\partial\, y^2_{n+i}} \right]^{-1} .$$

The error of forecast is computed in percentage, whenever the sample observed value of the dependent variable is available in the forecast periods :

$$\hat{e}_t = (1 - \hat{y}_t / y^0_t)\ 100 \quad , \quad (t=n+1, \ldots, n+p) \qquad (59)$$

where $\hat{y}_t$ and y^0_t are respectively the predicted and observed values of y_t .

The 95% confidence interval for y_t is also computed :

$$\hat{y}_t - 1.96\ \hat{\sigma}_t < y_t < \hat{y}_t + 1.96\ \hat{\sigma}_t \quad , \quad (t=n+1, \ldots, n+p) \qquad (60)$$

where $\hat{\sigma}_t$ is estimated from (56) and (57).

b) Simulation Forecast

In addition to the maximum likelihood predicted value, the simulated value of the dependent variable is computed as :

$$\tilde{y}_t = \left\{ \begin{array}{ll} \left[1 + \lambda_y \sqrt{f(X_t)} \left(\sum_{\ell} \rho_\ell \, y^*_{t-\ell} + \sum_k X^{**}_{kt} \beta_k \right) \right]^{1/\lambda_y} & (\lambda_y \neq 0) \\ \exp \left[\sqrt{f(X_t)} \left(\sum_{\ell} \rho_\ell \, \dfrac{\ln y_{t-\ell}}{\sqrt{f(X_t)}} + \sum_k X^{**}_{kt} \beta_k \right) \right] & (\lambda_y = 0) \end{array} \right\} \quad (t=n+1,\ldots,n+p) \qquad (61)$$

where the $y_{t-\ell}$ is replaced by $\tilde{y}_{t-\ell}$ if $t-\ell > n$.

For the case $\lambda_y \neq 0$, if the expression between the squared brackets happens to be negative, then it cannot be raised to the power $1/\lambda_y$: the program will be stopped at that point of the run.

The bias of $\tilde{y}_t$ with respect to the maximum likelihood predicted value, $\hat{y}_t$ when both $\tilde{y}_t$ and $\hat{y}_t$ are positive is :

- negative when $\lambda_y < 0$ or $\lambda_y > 1$,
- null when $\lambda_y = 1$,
- and positive when $0 < \lambda_y < 1$.

If the two roots of (52) are denoted by :

$$\hat{y}^{\lambda_y}_{n+1} = (- B + \sqrt{B^2 - 4C})/2 \text{ and } \hat{\hat{y}}^{\lambda_y}_{n+1} = (- B - \sqrt{B^2 - 4C})/2$$

where $\hat{y}^{\lambda_y}_{n+1}$ is the greater root, then it can be shown that :

$$\hat{y}^{\lambda_y}_{n+1} = \tilde{y}^{\lambda_y}_{n+1} \left(1 + \sqrt{1 - 4C/\tilde{y}^{2\lambda_y}_{n+1}}\right)/2 \qquad (62)$$

where $\tilde{y}^{\lambda_y}_{n+1} = - B$.

If $\lambda_y < 0$ or $\lambda_y > 1$, then C is negative, $\sqrt{1 - 4C/\tilde{y}^{2\lambda_y}_{n+1}} > 1$ and $\hat{y}^{\lambda_y}_{n+1} > \tilde{y}^{\lambda_y}_{n+1}$.

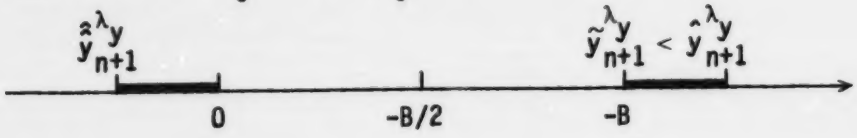
If $\lambda_y = 1$, then $C = 0$ and $\hat{y}_{n+1}^{\lambda_y} = \tilde{y}_{n+1}^{\lambda_y}$.

If $0 < \lambda_y < 1$: then C is positive, $\sqrt{1 - 4C/\tilde{y}_{n+1}^{2\lambda_y}} < 1$ and $\hat{\hat{y}}_{n+1}^{\lambda_y} < \tilde{y}_{n+1}^{\lambda_y}$.

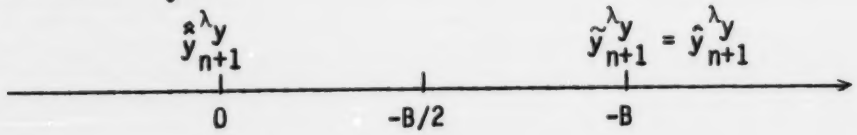
When $\lambda_y = 0$, it can readily inferred from (54) and (61) that $\hat{y}_{n+1} < \tilde{y}_{n+1}$.

Furthermore, since $\hat{y}_{n+1}^{\lambda_y} + \hat{\hat{y}}_{n+1}^{\lambda_y}$ for $\lambda_y \neq 0$, the bias $\tilde{y}_{n+1}^{\lambda_y} - \hat{y}_{n+1}^{\lambda_y}$ will be given by the smaller root $\hat{\hat{y}}_{n+1}^{\lambda_y}$. These results can be illustrated by the following diagrams :

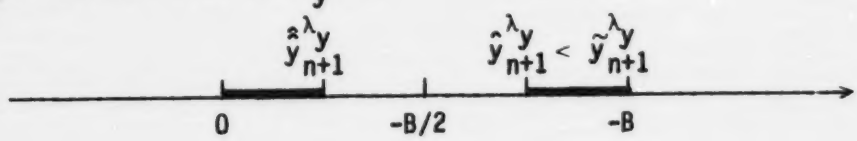
Case I : Negative bias ($\lambda_y < 0$ or $\lambda_y > 1$)



Case II : No bias ($\lambda_y = 1$)



Case III : Positive bias ($0 \leq \lambda_y < 1$)



Choosing the greater root $\hat{y}_{n+1}^{\lambda_y}$ will ensure that the predicted value of y_{n+1} is always positive, since the smaller root $\hat{\hat{y}}_{n+1}^{\lambda_y}$ can be negative or zero.

These results will hold for every period of forecast t = n+1,...,n+p.

c) Special Case without Box-Cox Transformation on the Dependent Variable

When the Box-Cox transformation is not used for the dependent variable (option NLY=-1), the formula (51) reduces to :

$$\frac{\partial \text{ Log } L_{n+1}}{\partial y_{n+1}} = - \frac{1}{\sigma_w^2 \sqrt{f(X_{n+1})}} \left(\frac{u_{n+1}}{\sqrt{f(X_{n+1})}} - \sum_{\ell} \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}} \right) \qquad (63)$$

where $u_{n+1} = y_{n+1} - \sum_k \beta_k X_{k,n+1}^{(\lambda_{xk})}$.

Setting this derivative equal to zero gives a unique maximum likelihood predicted value for y_{n+1} :

$$\hat{y}_{n+1} = \sum_k \beta_k X_{k,n+1}^{(\lambda_{xk})} + \sqrt{f(X_{n+1})} \sum_{\ell} \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}} \quad . \qquad (64)$$

The simulated value of the dependent variable will be identical to the maximum likelihood predicted value :

$$\begin{aligned} \tilde{y}_{n+1} &= \sqrt{f(X_{n+1})} \left(\sum_{\ell} \rho_\ell \, y^*_{n+1-\ell} + \sum_k X^{**}_{k,n+1} \beta_k \right) \\ &= \sqrt{f(X_{n+1})} \left[\left(\sum_{\ell} \rho_\ell (y^*_{n+1-\ell} - \sum_k X^*_{k,n+1-\ell} \beta_k \right) + \sum_k X^*_{k,n+1} \beta_k \right] \qquad (65) \\ &= \sum_k \beta_k X_{k,n+1}^{(\lambda_{xk})} + \sqrt{f(X_{n+1})} \sum_{\ell} \rho_\ell \frac{u_{n+1-\ell}}{\sqrt{f(X_{n+1-\ell})}} = \hat{y}_{n+1} \quad . \end{aligned}$$

The cross second derivatives of Log L_{n+p} with respect to $\Pi_{(p)}$ and y_{n+p} in this case remain the same as in the general case, with the following simplifications :

i) The factor $y_{n+p}^{\lambda_y - 1} / [\sigma_w^2 \sqrt{f(X_{n+p})}]$ common to all derivatives becomes $1/[\sigma_w^2 \sqrt{f(X_{n+p})}]$;

ii) The derivatives $\partial^2 \text{ Log } L_{n+p} / \partial y_{n+p-s} \partial y_{n+p}$ and $\partial^2 \text{ Log } L_{n+p} / \partial y_{n+p}^2$ reduce respectively to

$$[\rho_s / \sqrt{f(X_{n+p-s})}] / [\sigma_w^2 \sqrt{f(X_{n+p})}] \quad \text{and} \quad -1/[\sigma_w^2 f(X_{n+p})];$$

iii) and the derivative $\partial^2 \text{ Log } L_{n+p} / \partial \lambda_y \partial y_{n+p}$ is no longer necessary, since the Box-Cox parameter λ_y is not used in this case.

VII. Special Options

Although the program L-1.2 does not require any specific information about the type of the independent variables when using the first two options given below, a classification of these variables is necessary for the third option which runs on a seperate program called TABLEX, due to the recomputations of the elasticities for certain types of variables.

All independent variables must be classified into one of the following types :

Type 0 : Strictly positive and continuous variables,

Type 1 : All continuous variables that cannot be classified under Type 0,

Type 2 : Quasi-dummies (continuous variables containing only positive and null values),

Type 3 : Real dummies (binary variables representing temporal or spatial effects, and qualitative variables such as sex, marital status, social or occupational class, etc.).

Types 0 and 1 are used to distinguish respectively the variables which can be transformed by Box-Cox from those which cannot. Since they are both of continuous type, the associated elasticities evaluated from L-1.2 will not be recomputed in TABLEX. Type 2 is only a subgroup of Type 1, but the distinction allows the user to apply the Box-Cox transformation on the quasi-dummies with the first option described below. Moreover, the elasticities associated with these variables will be recomputed just as those for real dummies in TABLEX - hence their name "quasi-dummies" - by using the following approximation given in Dagenais, Gaudry and Liem (1985):

$$\eta_j^+ = \eta_j \, \bar{X}_j^+ / \bar{X}_j \qquad (66)$$

where η_j can be either $\eta_j^{(s)}$ or $\eta_j^{(e)}$ computed at the means of the dependent and independent variables for the whole estimation period, and $\bar{X}_j$ and $\bar{X}_j^+$ are the sample means of X_j computed respectively for the whole estimation period and for the subset of positive observations only.

1) Box-Cox Transformation on Quasi-Dummies

Strictly speaking, the variables of Type 1 cannot be transformed by Box-Cox, but those which contains only positive and null observations can be reclassified under Type 2 (Quasi-dummies) if the user is interested in applying the Box-Cox transformation to them in a special way :

i) The transformation is used only for the positive observations of the variable, leaving the null observations unaffected,

ii) a real dummy associated with this variable must be introduced into the regression equation to capture separately the effect of the null observations : thus this new variable has a value of one for the null observations of the quasi-dummy and a zero value otherwise.

Great care must be taken when several quasi-dummies are used simultaneously with this option, because of the multicollinearity problem associated with the newly created dummies.

2) Retrieval of the Estimation Results for Tabulation

This option allows the user to retrieve from TAPE1 the estimation results for each run of L-1.2, namely :

i) The name of the dependent variable, the code number of the run (NPASS), and other specifications for the equation being estimated : NR, NC, NF, NB, NRO, NSL, NGF, NGV, NLY, NDM, NLE, ND, where NR(=NF-NC+1) is the number of observations used for estimation;

ii) the derivatives $\partial y_t / \partial X_{jt}$ computed at the sample means of the variables in their original units and all the remaining parameters ρ, λ_x, λ_y, δ and λ_e (fixed or estimated);

iii) the unconditional and conditional t-values for β, ρ and δ;

iv) the unconditional t-values for λ_x, λ_y and λ_e under two null hypotheses (H_0: λ=0 and H_0: λ=1);

v) the four types of elasticities : two computed with the observations from the whole estimation period (sample and expected values of y) and two estimated from the last portion of the estimation period (sample and expected values of y);

vi) the log-likelihood and pseudo-(L)-R^2 values, and the average probability of y to be at the limit computed in (31).

For the CDC-CYBER 835/855 installed at the Université de Montréal, the results can be stored in a file to be used as input to the program TABLEX. After the LGO card for each run of L-1.2 (see Section VIII), simply add :

```
REWIND (TAPE1)
LIB (C,TAPE1,N=file name)
```

.

3) Tabulation : Program TABLEX

This third option, available on a separate program called TABLEX, is designed to report the estimation results of several equations in a table which can be used directly in research papers or review articles.

The input of TABLEX contains four parts :

I) The listing options;

II) The definitions of the independent variables which are grouped into blocks;

III) The results of the equations estimated from L-1.2;

IV) The observations for the quasi and real dummies.

Part I : The Listing Options

```
_$CARD_NBLOC=..,NEQ=..,NEL=..,NSTU=..,NPD=..,NDT=..,NVS=..,NLIST=..,NOBS=..$
(NAMD(I),I=1,NDT) : necessary only if NDT> 0 (FORMAT(5A8)).
```

where

NBLOC : Number of blocks of independent variables (max. 15). The blocks are used to classify the independent variables into a few broad categories. For example, in transportation demand analysis, some typical blocks can be groups of variables representing the characteristics of motor vehicles, transportation networks, consumers, prices, and so forth.

NEQ : Number of equations to be listed (max. 9).

NEL : Control parameter for the listing of the derivatives $\partial y_t / \partial X_{jt}$ or the elasticities η.

NEL = 0 : List the derivatives $\partial y_t / \partial X_{jt}$ computed at the sample means of the variables in their original units. Since the derivative with respect to the constant is not computed, it is set equal to zero in the table,

1 : list the elasticities $\eta^{(s)}$ computed at the sample means of the dependent and independent variables for the whole estimation period,

2 : list the eleasticities $\eta^{(e)}$ computed from E(y) evaluated at the sample means of the independent variables for the whole estimation period,

3 and 4 : same as 1 and 2 respectively, but for the last portion of the estimation period; the elasticities for the quasi and real dummies are not recomputed according to (66), unlike for NEL=1,2.

NSTU : Control parameter for the listing of the t-values for β, ρ, λ_y, λ_x, δ and λ_e. In the table, all t-values are enclosed in parentheses and those for λ_y, λ_x and λ_e are listed under H_o: λ=0 (first row) and H_o: λ=1 (second row).

NSTU = 0 : List the unconditional t-values for all the parameters,

1 : list the conditional t-values for β, ρ and δ, and the unconditional t-values for λ_y, λ_x and λ_e.

NPD : Control parameter for the recomputation of the elasticities associated with the quasi-dummies.

NPD = 0 : The elasticities are recomputed according to (66) for the quasi-dummies which are classified under Type 2,

1 : the elasticities are not recomputed, i.e. the quasi-dummies are considered in this case as a subgroup of Type 1, although they are coded as Type 2.

NDT : Number of variables used for heteroskedasticity in the NEQ equations (max. 10). A variable must be counted only once even if it appears in several equations.

NVS : Control parameter for the language version in which the table is edited.

NVS = 0 : The table is listed in French or in any other language preferred by the user.

1 : The table is listed in English.

NLIST : Control parameter for the listing of the definitions of the independent variables, before the table is reproduced.

NLIST = 0 : Only the table is listed (normal run),

1 : Only the list of definitions is printed (option used for checking if the list contains all the definitions of the independent variables appearing in the NEQ equations, including the variables used for heteroskedasticity).

NOBS : Total number of observations (max. 999) which will be read in Part IV, for the quasi and real dummies.

NAMD : Name of the variable used for heteroskedasticity. If several variables are used, their entry order will determine the listing order in the table.

Part II : The Definitions of the Independent Variables

The list of definitions for the independent variables must be as complete as possible, not only for the particular equations being reported but also for the whole model under study. The variables (max. 500) can be grouped into blocks (max. 15) and must include explanatory variables used for regression as well as heteroskedasticity. By convention, the name for the regression constant must be declared as CONSTANT, since L-1.2 stores the results associated with this variable under that name.

The input field is identical for the blocks and the independent variables : FORMAT(I2,A8,3A10,A5,3A10,A5). Within each block, any number of variables can be entered and their entry order will determine the listing order in the table. A blank card is used to mark the end of a block.

```
Index number,Name,Definition of the block in two languages \ for NVS=0 and 1
Type number,Name,Definiton of the variable in two languages / respectively
        .
        .
        .
Blank card
```

Repeat the sequence for NBLOC blocks.

Note that the quasi and real dummies (Types 2 and 3) are underlined respectively with --- and === in the output table. When NPD=1, the quasi-dummies are not underlined, since their elasticities are not recomputed.

Part III : The Results of the Equations Estimated from L-1.2

The results of the NEQ equations must appear in this part. Their entry order will determine the listing sequence of the columns associated with the equations in the table. No input format is given since TABLEX will read the results exactly in the same format as when they were stored by L-1.2.

Part IV : The Observations for the Quasi and Real Dummies

This part is only necessary if NEL= 1 or 2 and if the list of variables in Part II contains quasi and real dummies. It is used to recompute the elasticities associated with these variables by the formula (66). All the quasi and real dummies in the list of Part II must appear here and must follow the entry order specified in the list.

Fortran format within parameters for the name and the observations of the dummy (80 columns) Name, Observations of the first dummy . . . Name, Observations of the last dummy

VIII. Sample Deck Structure and Print-Out

For illustrative purposes, the transit demand equation for adults in the DEMTEC model (Dagenais, Gaudry and Liem, 1985) is estimated with L-1.2. The total sample runs from December 1956 to December 1971 (NOBS=181). Since the first and twelfth autocorrelation orders are considered (NRO=2) and the three last months of the sample are used for forecasting, the number of observations available for estimation is 166 (NC=13, NF=178 and NFP=181). The elasticities are computed for the whole period of estimation as well as for the last twelve months of the period (NOBEL=12).

The dependent variable is XCANFA and the independent variables (NB=27) are split into two groups :

i) group without Box-Cox transformation (NSL=14) : TLK, RFK, SFK, CSFMR, RCUTP, ANT, ANTL1, STR, EX7, EX8, EX9, EX0, EX1 and DUMZA;

ii) group with a common estimated λ_x (NGV=1, MAXV=13) : FAD, NC, P, RAWEM, TW, TT T2T, EANFP, SHONF, WD, SAT, SHD and ZA.

Note that ZA is a quasi-dummy on which we wish to apply the Box-Cox transformation in a special way, i.e. by introducing into the equation an associated real dummy, DUMZA. The dependent variable is transformed by Box-Cox with an estimated λ_y (NLY=1) and finally the independent variable SHONF is also used in the heteroskedasticity function $f(X_t)$ where both λ_e and δ are estimated (NDM=1, NLE=1 and ND=1).

The regression constant is generated by the program itself and has a reserved name "CONSTANT". In the print-out, the type of λ_{xk} associated with each independent variable is indicated by :

NL = "No Lambda" for the group without Box-Cox transformation (The group is numbered as "0" : the constant is automatically assigned to this group);

FL = "Fixed Lambda" for the NGF groups;

EL = "Estimated Lambda" for the NGV groups.

To reduce the output listing, only this options NCOR and NPDQL are activated.

SAMPLE DECK A

(as executed on a CDC-CYBER 835/855
under NOS/BE Version 1.5)

```
USER,CM210000,T500.                          }
*CODE                                        }  Control Statements
FTN(L=0)                                     }
LGO.
*WEOR
   .
   .
Fortran Source Deck Containgin Program L-1.2
   .
   .
*WEOR
_$CARD1_NOBS=181,NC=13,NF=178,NFP=181,NOBEL=12,NM=0,NSTEP=500,EPSI=1.E-7,WGHT=1.$
_$CARD2_NB=27,NRO=2,NSL=14,NGF=0,MAXF=0,NGV=1,MAXV=13,NLY=1,NDM=1,NLE=1,ND=1$
_$CARD3_NCOR=2,NRES=0,NPW=0,NGUVW=0,NPDQL=2,NCOV=0,NSTORE=0,NGRID=0,NPASS=1,NLSQ=0$

_AUTOCORRELATION(1,12) - 1 LAMBDA(X) - 1 LAMBDA(Y) - 1 DELTA - 1 LAMBDA(E)
1_12_0._0.                                   }
1._13                                        }  Values are entered in free format, i.e.
1._0                                         }  separated by at least one blank denoted
0._1._23                                     }  as _ .
0_0_12_36_36                                 }
(3XA8,54XE15.7/(20X4E15.7))
Name and Observations for XCANFA (Dependent Variable)
            .              TLK               }
            .               .                }  Group without Box-Cox transformation
            .               .                }        (14 variables)
            .              DUMZA             }
            .              FAD               }
            .               .                }  Group with a common estimated λx
            .               .                }        (13 variables)
Name and observations for ZA
```

To save the compilation time, the program L-1.2 can be first stored on FILE1 in its compiled form. If the model contains a great number of variables to be accessed to randomly, the UPDATE program available on the CDC-CYBER can be used. Suppose that all the variables are previously stored on FILE2 in UPDATE format, then the sample deck appears as follows :

SAMPLE DECK B

```
USER,CM210000,T500.                          )
*CODE                                        |
LIB(L,LGO,N=FILE1)                           |
COPYCR(INPUT,DATUM)                          |
LIB(L,OLDPL,N=FILE2)                         |
UPDATE(K,Q,D,8,L=0)                          } Control Statements
COPYCR(COMPILE,DATUM)                        |
REWIND(DATUM)                                |
COMBINE(DATUM,DATA,2)                        |
REWIND(DATA)                                 |
LGO(DATA)                                    |
*WEOR                                        )
_$CARD1_.........................$           )
    .                                        |
    .                                        } Same input cards as in SAMPLE DECK A
    .                                        |
(3XA8,54XE15.7/(20X4E15.7))                  )
*WEOR
*C_XCANFA
*C_TLK,RFK,SFK,CSFMR,RCUTP,ANT,ANTL1,STR,EX7,EX8,EX9,EX0,EX1,DUMZA
*C_FAD,NC,P,RAWEM,TW,TT,T2T,EANFP,SHONF,WD,SAT,SHD,ZA
```

The sample decks A and B are illustrated without the NGRID option. To use this option, one must proceed in two steps :

1) Run normally the program with NSTORE=1 and NGRID=0, and save the function and parameters values from TAPE3 on a file named MAXLOG; simply add in the control statements section, after the LGO card :

```
REWIND(TAPE3)
LIB(C,TAPE3,N=MAXLOG)
```

Note that unformatted (binary) READ or WRITE is used on TAPE3.

2) To obtain a grid for β_1 and β_4, for example, card type no. (16) must be added at the end of the data section :

```
*WEOR                    (This card is necessary
                          for sample deck B only)
BETA_1
BETA_4
BE_1_0.25                (Sample decks A and B)
EL_1
BE_4_0.025
EL_2
```

Run the program with NSTORE=2 and NGRID=1. A slight modification in the control statements section is required. The two cards after the LGO card are no longer useful and they must be deleted. Two new cards are required to connect the file MAXLOG to the program, after FTN card in sample deck A or after LIB(L,LGO,N=FILE1) in sample deck B :

```
LIB(L,TAPE3,N=MAXLOG)
REWIND(TAPE3)
```

For sample deck B, two additional modifications are needed :

i) After COPYCR(COMPILE,DATUM), insert COPYCR(INPUT,DATUM);

ii) Replace the number 2 with 3 in COMBINE(DATUM,DATA,2).

PRINT-OUT

```
$CARD1
NOBS    = 181,
NC      = 13,
NF      = 178,
NFP     = 181,
NOBEL   = 12,
NM      = 0,
NSTEP   = 500,
EPSI    = .1E-06,
WGHT    = .1E+01,
$END
```

```
$CARD2
NB      = 27,
NRO     = 2,
NSL     = 14,
NGF     = 0,
MAXF    = 0,
NGV     = 1,
MAXV    = 13,
NLV     = 1,
NDM     = 1,
NLE     = 1,
ND      = 1,
$END
```

```
$CARD3
NCOR    = 2,
NRFS    = 0,
NPW     = 0,
NGUVW   = 0,
NPDQL   = 2,
NCOV    = 0,
NSTORE  = 0,
NGRID   = 0,
NPASS   = 1,
NLSQ    = 0,
$END
```

```
WORKING SPACE USED : 13883

AUTOCORRELATION OF ORDERS 1 12
INITIAL VALUES FOR RHOS   0. 0.

GROUP (X) WITHOUT BOX-COX TRANSFORMATION
VARIABLE NO. 1 2 3 4 5 6 7 8 9 10 11 12 13 14

GROUP 1   INITIAL VALUE FOR LAMBDA(X) 1.
VARIABLE NO. 15 16 17 18 19 20 21 22 23 24 25 26 27

INITIAL VALUE FOR LAMBDA(Y) 1.
UPPER LIMIT FOR Y 0.

HETEROSCEDASTICITY
INITIAL VALUES FOR DELTAS 0.
INITIAL VALUES FOR LAMBDAS(E) 1.
VARIABLE NO. 23
```

	MEANS OF ORIGINAL VARIABLES COMPUTED FROM THE SUBSET OF POSITIVE OBSERVATIONS		NUMBER OF POSITIVE OBSERVATIONS	SCALING FACTORS (POWERS OF 10)
	XCANFA	.2033392E+08	181	7
1	TLK	.1946111E+03	180	2
2	RFK	.7225757E+00	179	0
3	SFK	.4913224E+01	96	0
4	CSFMR	.4655368E+02	95	1
5	RCUTP	.2384263E+05	172	4
6	ANT	.1000000E+01	1	0
7	ANTL1	.1000000E+01	1	0
8	STR	.1466667E+02	3	1
9	EX7	.2611922E+07	7	6
10	EX8	.6866020E+06	6	5
11	EX9	.5944373E+06	4	5
12	EX0	.5145238E+06	4	5
13	EX1	.4598918E+06	4	5
14	DUMZA	.1000000E+01	73	0
15	FAD	.1634673E+00	181	0
16	NC	.3048819E+06	181	5
17	P	.1393585E+03	181	2
18	RAWEM	.7874157E+02	181	1
19	TW	.3919595E+01	181	0
20	TT	.2147617E+02	181	1
21	12T	.1001378E+02	181	1
22	EANFP	.1241095E+09	181	8
23	SHONF	.2099588E+06	181	5
24	WD	.2081768E+02	181	1
25	SAT	.4348066E+01	181	0
26	SHD	.5270718E+01	181	0
27	ZA	.8098551E-01	108	-1

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS									
			TLK	PFK	SFK	CSFMR	RCUTP	ANT	ANTL1	STR	EX7	EX8
1	.0000	966.96	.0014	.0000	.0000	.0001	.0030	.0011	.0015	.0008	.0024	.0053
2	.0001	352.61	.0052	.0012	.0099	.0000	.0082	.0024	.0040	.0072	.1101	.0177
3	.0001	337.52	.2393	.0045	.0093	.0272	.0246	.0007	.0038	.0110	.0096	.0972
4	.0005	175.24	.0058	.0078	.0342	.0086	.0037	.0148	.0000	.0047	.0149	.0133
5	.0007	150.58	.0023	.0026	.0007	.0016	.0118	.0140	.0410	.0030	.1359	.0057
6	.0013	114.99	.0197	.0085	.0605	.0002	.0444	.0007	.0000	.0043	.0304	.0031
7	.0015	105.90	.0042	.0036	.0046	.0002	.0117	.0086	.0155	.0075	.0021	.0225
8	.0060	52.51	.0017	.0043	.0130	.0012	.0158	.0010	.0063	.0041	.0202	.0008
9	.0080	45.50	.0103	.0001	.0052	.0020	.1334	.0008	.0005	.0037	.0188	.0091
10	.0104	40.09	.0000	.0005	.0006	.0036	.0482	.0004	.0091	.0043	.0020	.0009
11	.0185	29.96	.0239	.0028	.0102	.0124	.0106	.0203	.0001	.0007	.0018	.0002
12	.0227	27.10	.0022	.0027	.0049	.0774	.0864	.0079	.0019	.0004	.0073	.0237
13	.0432	19.63	.0081	.0317	.1707	.2580	.0017	.0050	.0049	.0000	.0073	.0325
14	.0796	14.46	.5977	.0863	.1583	.1161	.0729	.0007	.0032	.0059	.0216	.0373
15	.1167	11.94	.0682	.0848	.0483	.0063	.4443	.0011	.0046	.0010	.1275	.1075
16	.1621	10.14	.0000	.7360	.0475	.0653	.0359	.0199	.0051	.0163	.0128	.0061
17	.1863	9.45	.0018	.0013	.3716	.3808	.0010	.0380	.0033	.0000	.0011	.0002
18	.5934	5.30	.0025	.0025	.0221	.0127	.0358	.0034	.0038	.0186	.0633	.1439
19	.7883	4.60	.0000	.0005	.0000	.0000	.0010	.0005	.0000	.5787	.2460	.0079
20	.9697	4.14	.0000	.0015	.0001	.0007	.0009	.1247	.6961	.0000	.0009	.0004
21	.9961	4.09	.0001	.0000	.0018	.0000	.0001	.6968	.1298	.0000	.0000	.0085
22	1.0004	4.08	.0000	.0000	.0000	.0000	.0000	.0038	.0058	.0000	.0000	.0157
23	1.0015	4.08	.0000	.0001	.0001	.0000	.0000	.0221	.0000	.0000	.0000	.0286
24	1.0032	4.07	.0000	.0002	.0001	.0000	.0001	.0000	.0169	.0000	.0002	.3568
25	1.1752	3.76	.0000	.0004	.0001	.0001	.0000	.0024	.0002	.3076	.1340	.0314
26	1.3310	3.54	.0012	.0097	.0065	.0066	.0034	.0026	.0427	.0045	.0121	.0085
27	1.8341	3.01	.0040	.0061	.0194	.0188	.0009	.0063	.0000	.0157	.0175	.0154
28	16.6493	1.00	.0002	.0004	.0002	.0002	.0003	.0000	.0000	.0001	.0001	.0001

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS									
			EX9	EX0	EX1	DUMZA	FAD	NC	P	RAWEM	TW	TT
1	.0000	966.96	.0084	.0216	.0328	.0045	.2165	.7913	.9548	.0821	.0258	.0229
2	.0001	352.61	.0016	.0005	.0012	.3632	.0130	.0543	.0080	.2976	.0070	.3138
3	.0001	337.52	.0777	.0011	.0482	.0161	.0140	.0617	.0355	.6073	.0110	.0059
4	.0005	175.24	.0276	.0025	.0017	.2576	.0963	.0363	.0006	.0046	.0075	.0830
5	.0007	150.58	.0030	.0025	.0074	.0835	.0143	.0278	.0007	.0027	.9325	.0172
6	.0013	114.99	.0073	.0431	.0831	.1915	.0001	.0070	.0001	.0025	.0000	.3679
7	.0015	105.90	.0015	.0152	.0777	.0622	.4849	.0112	.0003	.0027	.0059	.1469
8	.0060	52.51	.0152	.0155	.0129	.0072	.0766	.0072	.0000	.0002	.0049	.0005
9	.0080	45.50	.0281	.0157	.0053	.0101	.0632	.0006	.0000	.0000	.0010	.0352
10	.0104	40.09	.0143	.0101	.0060	.0015	.0162	.0002	.0000	.0000	.0001	.0021
11	.0185	29.96	.0026	.0006	.0016	.0000	.0001	.0000	.0000	.0000	.0001	.0008
12	.0227	27.10	.0527	.0976	.1046	.0001	.0030	.0023	.0000	.0001	.0035	.0033
13	.0432	19.63	.0318	.0442	.0322	.0003	.0019	.0001	.0000	.0001	.0006	.0000
14	.0796	14.46	.0078	.0129	.0148	.0002	.0000	.0000	.0000	.0000	.0000	.0001
15	.1167	11.94	.0849	.0986	.0612	.0012	.0000	.0000	.0000	.0000	.0000	.0001
16	.1621	10.14	.0302	.0070	.0123	.0002	.0000	.0000	.0000	.0000	.0000	.0000
17	.1863	9.45	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
18	.5934	5.30	.0855	.0916	.0730	.0002	.0000	.0000	.0000	.0000	.0000	.0000
19	.7883	4.60	.0084	.0067	.0056	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20	.9697	4.14	.0158	.0001	.0026	.0000	.0000	.0000	.0000	.0000	.0000	.0000
21	.9961	4.09	.0096	.0020	.0052	.0000	.0000	.0000	.0000	.0000	.0000	.0000
22	1.0004	4.08	.0001	.3598	.1833	.0000	.0000	.0000	.0000	.0000	.0000	.0000
23	1.0015	4.08	.2715	.1030	.1504	.0000	.0000	.0000	.0000	.0000	.0000	.0000
24	1.0032	4.07	.1731	.0035	.0420	.0000	.0000	.0000	.0000	.0000	.0000	.0000
25	1.1752	3.76	.0264	.0264	.0219	.0000	.0000	.0000	.0000	.0000	.0000	.0000
26	1.3310	3.54	.0023	.0065	.0028	.0003	.0000	.0000	.0000	.0000	.0000	.0000
27	1.8341	3.01	.0123	.0115	.0100	.0000	.0000	.0000	.0000	.0000	.0000	.0000
28	16.6493	1.00	.0000	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS							
			T2T	EANFP	SHONF	WD	SAT	SHD	ZA	CONSTANT
1	.0000	966.96	.1873	.0039	.0036	.0290	.0000	.0182	.0008	.8086
2	.0001	352.61	.6062	.1064	.0427	.0536	.0183	.0036	.3584	.1194
3	.0001	337.52	.0165	.0105	.0780	.0552	.0087	.0565	.0190	.0666
4	.0005	175.24	.1308	.0058	.0917	.6156	.2520	.2689	.2455	.0048
5	.0007	150.58	.0001	.2037	.0056	.0024	.0087	.0022	.1020	.0001
6	.0013	114.99	.0120	.0268	.0150	.1609	.0213	.0615	.1881	.0000
7	.0015	105.90	.0378	.0024	.0004	.0492	.0260	.0117	.0698	.0001
8	.0060	52.51	.0084	.2063	.1158	.0170	.0038	.0346	.0106	.0002
9	.0080	45.50	.0001	.2103	.0001	.0129	.1103	.0101	.0018	.0001
10	.0104	40.09	.0005	.0262	.0251	.0003	.4994	.1691	.0001	.0000
11	.0185	29.96	.0000	.0773	.0641	.0033	.0441	.3284	.0003	.0000
12	.0227	27.10	.0001	.1156	.1235	.0002	.0056	.0198	.0023	.0000
13	.0432	19.63	.0001	.0020	.4073	.0003	.0005	.0127	.0001	.0000
14	.0796	14.46	.0000	.0023	.0072	.0000	.0003	.0002	.0000	.0000
15	.1167	11.94	.0000	.0001	.0010	.0000	.0007	.0014	.0004	.0000
16	.1621	10.14	.0000	.0001	.0026	.0000	.0001	.0007	.0004	.0000
17	.1863	9.45	.0000	.0000	.0157	.0000	.0001	.0003	.0000	.0000
18	.5934	5.30	.0000	.0000	.0005	.0000	.0000	.0000	.0001	.0000
19	.7883	4.60	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20	.9697	4.14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
21	.9961	4.09	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
22	1.0004	4.08	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
23	1.0015	4.08	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
24	1.0032	4.07	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
25	1.1752	3.76	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
26	1.3310	3.54	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0000
27	1.8341	3.01	.0000	.0000	.0001	.0000	.0000	.0000	.0000	.0000
28	16.6493	1.00	.0000	.0000	.0001	.0000	.0000	.0000	.0000	.0000

CORRELATION MATRIX

TLK	1.0000														
RFK	-.6005	1.0000													
SFK	.7900	-.5507	1.0000												
CSFMR	.7668	-.5961	.7351	1.0000											
RCUTP	.3562	-.3391	.3172	.3652	1.0000										
ANT	.0522	-.1126	-.0167	.1055	.0385	1.0000									
ANTL1	-.0179	.0182	-.0543	-.0163	.0246	-.0061	1.0000								
STR	-.1147	-.0155	-.0910	-.0950	.0152	-.0102	-.0102	1.0000							
EX7	-.1268	.0374	-.1246	-.1337	-.1616	-.0144	-.0144	.2063	1.0000						
EX8	-.1529	.0349	-.1252	-.1307	-.1118	-.0140	-.0140	-.0234	-.0332	1.0000					
EX9	-.0897	.1228	-.1012	-.1056	-.1256	-.0113	-.0113	-.0189	-.0268	-.0261	1.0000				
EX0	-.0914	.0451	-.1039	-.1085	-.1157	-.0116	-.0116	-.0195	-.0276	-.0268	-.0216	1.0000			
EX1	-.1133	.0858	-.0989	-.1032	-.1124	-.0110	-.0110	-.0185	-.0262	-.0255	-.0206	-.0211	1.0000		
DUMZA	.0242	-.0688	-.0448	-.0098	.4321	-.0665	-.0665	.0688	.2165	.2104	.1699	.1746	.1661	1.0000	
FAD	-.0172	-.0197	-.0651	-.0469	.4455	-.0607	-.0140	.0827	.1941	.1592	.2527	.2433	.2152	.8596	1.0000
NC	-.0268	-.0255	-.0816	-.0655	.4882	-.0061	-.0053	.0611	.1097	.1848	.1896	.2339	.2583	.8605	.9090
P	-.0141	-.0341	-.0643	-.0523	.4789	-.0234	-.0209	.0552	.1218	.1816	.2127	.2514	.2732	.8894	.9441
RAWEM	-.0695	-.0139	-.1010	-.0541	.4913	-.0059	-.0042	.0471	.0891	.1466	.1518	.2402	.3122	.8441	.8977
TW	-.1073	.0787	-.1638	-.1518	.3577	-.0449	-.0359	.0358	-.0044	.1685	.2068	.2846	.2736	.7435	.8236
TT	.0757	-.0534	.1404	.1404	-.2810	.0503	.0494	-.0878	-.2648	-.2505	-.2152	-.2145	-.2043	-.7209	-.8563
T2T	-.0021	-.0496	.0467	.0225	.3688	-.0758	-.0765	-.0446	-.0300	.0845	.1674	.2824	.3835	.6208	.8055
EANFP	.0840	-.1705	.1487	.1509	.4122	.0177	.0254	.1136	-.0068	-.0687	-.1434	-.1935	-.2487	.1467	.0340
SHONF	-.0705	.0342	.0175	-.2564	.2927	-.0275	.0179	.0696	.0774	.0187	.0025	.0241	.0416	.3935	.3339
WD	-.1216	.0701	-.1497	-.0169	-.1573	.1673	-.0635	-.0068	.0874	.0565	.0400	.0726	.0757	.0028	.0068
SAT	-.0617	.0616	-.0723	-.0820	-.0707	-.0563	-.0563	-.0319	-.0012	.0373	.0088	-.0052	.0040	-.0009	-.0053
SHD	-.0353	.0231	-.0319	-.1972	.0409	-.1208	.0702	.0303	-.0511	-.0411	-.0098	-.0485	-.0461	-.0083	-.0163
ZA	-.0525	.0963	.0088	-.0226	-.4298	.0727	.0820	-.0631	-.2148	-.2087	-.1686	-.1732	-.1648	-.9922	-.8452
XCANFA	.0915	-.1240	.0996	.1420	-.0902	0413	.0310	-.5016	.5457	.0574	-.0894	-.1389	-.1250	-.0533	-.1335

	TLK	RFK	SFK	CSFMR	RCUTP	ANT	ANTL1	STR	EX7	EX8	EX9	EX0	EX1	DUMZA	FAD

CORRELATION MATRIX

	NC	P	RAWEM	TW	TT	T2T	EANFP	SHONF	WD	SAT	SHD	ZA	XCANFA
NC	1.0000												
P	.9900	1.0000											
RAWEM	.9833	.9758	1.0000										
TW	.9325	.9105	.9274	1.0000									
TT	-.7717	-.8034	-.7450	-.6879	1.0000								
T2T	.7018	.7644	.7356	.6398	-.7467	1.0000							
EANFP	.0148	.0209	.0047	-.1856	.0636	-.0783	1.0000						
SHONF	.4165	.4017	.3450	.3331	-.2491	.1703	.2914	1.0000					
WD	.0154	.0091	.0344	.0221	-.0551	.0113	-.1905	-.1651	1.0000				
SAT	.0020	.0017	.0034	.0800	-.0077	-.0148	-.0940	.0527	-.4519	1.0000			
SHD	-.0179	-.0165	-.0642	.0027	.0130	-.0125	.0545	.4081	-.5865	.2162	1.0000		
ZA	-.8176	-.8583	-.8029	-.6853	.7133	-.6439	-.1706	-.3716	.0044	.0151	.0093	1.0000	
XCANFA	-.2025	-.1851	-.2164	-.3519	-.0021	-.1729	.1815	.0487	.1426	-.0334	-.0331	.0256	1.0000

EQUATION : XCANFA

	VARIABLE	LAMBDA(X) TYPE	GROUP	COEFFICIENT	MEAN(X)	STD-ERROR	STUDENT-T	ELAST(S) (OBS.NO.	ELAST(E) 13- 178)	ELAST(S) (OBS.NO.	ELAST(E) 167- 178)
1	TLK	NL	0	-.2878423E-02	.1931928E+01	.6421526E-02	-.4482	-.0027	-.0026	-.0031	-.0030
2	RFK	NL	0	-.3073578E-01	.7177468E+00	.1135630E-01	-2.7065	-.0109	-.0103	-.0091	-.0098
3	SFK	NL	0	-.3245352E-02	.2659354E+01	.2167703E-02	-1.4971	-.0043	-.0040	-.0072	-.0072
4	CSFMR	NL	0	.1495132E-01	.2512711E+01	.2406014E-02	6.2141	.0186	.0176	.0278	.0278
5	RCUIP	NL	0	.1188649E-01	.2281015E+01	.4468477E-02	2.6601	.0134	.0127	.0269	.0288
6	ANT	NL	0	-.1042916E+00	.6024096E-02	.5670343E-01	-1.8392	-.0003	-.0003	0.0000	0.0000
7	ANTL1	NL	0	.7318308E-01	.6024096E-02	.5617639E-01	1.3027	.0002	.0002	0.0000	0.0000
8	STR	NL	0	-.7305231E+00	.2650602E-01	.2155516E-01	-33.8909	-.0096	-.0091	0.0000	0.0000
9	EX7	NL	0	.2522181E+00	.1101413E+00	.1001767E-01	25.1773	.0137	.0130	0.0000	0.0000
10	EX8	NL	0	.2489494E-01	.2481694E+00	.3785137E-02	6.5770	.0031	.0029	0.0000	0.0000
11	EX9	NL	0	.1833634E-01	.1432379E+00	.5304174E-02	3.4570	.0013	.0012	0.0000	0.0000
12	EX0	NL	0	.1321741E-01	.1239816E+00	.6269677E-02	2.1081	.0008	.0008	0.0000	0.0000
13	EX1	NL	0	.1416604E-01	.1108173E+00	.7406362E-02	1.9127	.0008	.0007	.0114	.0114
14	DUMZA	NL	0	-.2774511E-01	.4216867E+00	.2985852E-01	-.9292	-.0058	-.0055	-.0148	-.0148
15	FAD	EL	1	-.1378218E+01	.1652917E+00	.5126762E+00	-2.6883	-.1126	-.1067	-.1621	-.1620
16	NC	EL	1	-.2141483E+00	.3110136E+01	.8519505E-01	-2.5136	-.3292	-.3118	-.5170	-.5167
17	P	EL	1	.5373867E+00	.1401901E+01	.5614218E+00	.9572	.3723	.3527	.4591	.4596
18	RAWEM	EL	1	.1645337E+00	.7928325E+01	.4054337E-01	4.0582	.6447	.6108	.8541	.8542
19	TW	EL	1	-.1573042E+00	.4003221E+01	.3373259E-01	-4.6633	-.3112	-.2948	-.4081	-.4079
20	TT	EL	1	-.4608212E+00	.2143780E+01	.5278483E-01	-8.7302	-.4882	-.4625	-.4631	-.4633
21	T2T	EL	1	-.8427340E+00	.9928416E+00	.2324426E+00	-3.6256	-.4135	-.3917	-.5121	-.5124
22	EANFP	EL	1	.2272498E+00	.1235222E+01	.3609216E-01	6.2964	.1387	.1314	.1391	.1396
23	SHONF	EL	1	.7220509E-01	.2104270E+01	.1106389E-01	6.5262	.0751	.0711	.0931	.0935
24	WD	EL	1	.5506692E+00	.2082530E+01	.6247970E-01	8.8136	.5668	.5369	.6049	.6045
25	SAT	EL	1	.3796268E-01	.4343373E+01	.1056678E-01	3.5926	.0815	.0772	.0861	.0867
26	SHD	EL	1	.3767269E-01	.5265060E+01	.7556238E-02	4.9856	.0980	.0929	.1043	.1042
27	ZA	EL	1	-.2666424E+00	.4750768E+00	.1717535E+00	-1.5525	-.0626	-.0593	0.0000	0.0000
28	CONSTANT	NL	0	-.1118994E+01	.1000000E+01	.5529500E+00	-2.0237	-.5530	-.5239	-.5901	-.5897

DERIVATIVES OF Y AND E(Y) WITH RESPECT TO THE INDEPENDENT VARIABLES (ORIGINAL UNITS)

	VARIABLE	DERIVATIVE(S) (OBS.NO.	DERIVATIVE(E) 13- 178)	DERIVATIVE(S) (OBS.NO.	DERIVATIVE(E) 167- 178)
1	TLK	-.2878423E+03	-.2878422E+03	-.2878423E+03	-.2878422E+03
2	RFK	-.3073578E+06	-.3073577E+06	-.3073578E+06	-.3073577E+06
3	SFK	-.3245352E+05	-.3245350E+05	-.3245352E+05	-.3245350E+05
4	CSFMR	.1495132E+05	.1495131E+05	.1495132E+05	.1495131E+05
5	RCUTP	.1188649E+02	.1188649E+02	.1188649E+02	.1188649E+02
6	ANT	-.1042916E+07	-.1042916E+07	-.1042916E+07	-.1042916E+07
7	ANTL1	.7318308E+06	.7318303E+06	.7318308E+06	.7318303E+06
8	STR	-.7305231E+06	-.7305227E+06	-.7305231E+06	-.7305227E+06
9	EX7	.2522181E+01	.2522180E+01	.2522181E+01	.2522180E+01
10	EX8	.2489494E+01	.2489493E+01	.2489494E+01	.2489493E+01
11	EX9	.1833634E+01	.1833633E+01	.1833634E+01	.1833633E+01
12	EX0	.1321741E+01	.1321740E+01	.1321741E+01	.1321740E+01
13	EX1	.1416604E+01	.1416604E+01	.1416604E+01	.1416604E+01
14	DUMZA	-.2774511E+06	-.2774509E+06	-.2774511E+06	-.2774509E+06
15	FAD	-.1378218E+08	-.1378217E+08	-.1378218E+08	-.1378217E+08
16	NC	-.2141483E+02	-.2141482E+02	-.2141483E+02	-.2141482E+02
17	P	.5373867E+05	.5373864E+05	.5373867E+05	.5373864E+05
18	RAWEM	.1645337E+06	.1645336E+06	.1645337E+06	.1645336E+06
19	TW	-.1573042E+07	-.1573041E+07	-.1573042E+07	-.1573041E+07
20	TT	-.4608212E+06	-.4608209E+06	-.4608212E+06	-.4608209E+06
21	T2T	-.8427340E+06	-.8427335E+06	-.8427340E+06	-.8427335E+06
22	EANFP	.2272498E-01	.2272497E-01	.2272498E-01	.2272497E-01
23	SHONF	.7220509E+01	.7220505E+01	.7220509E+01	.7220505E+01
24	WD	.5506692E+06	.5506688E+06	.5506692E+06	.5506688E+06
25	SAT	.3796268E+06	.3796266E+06	.3796268E+06	.3796266E+06
26	SHD	.3767269E+06	.3767266E+06	.3767269E+06	.3767266E+06
27	ZA	-.2666424E+08	-.2666423E+08	0.	0.

MEAN PROBABILITY OF Y TO BE AT THE LIMIT = 0.0000

MEAN (Y) : 2.0233914

E(V) AT THE MEANS	:	2.1358297
MEAN (V**)	:	1.0233914
PSEUDO-(E)-R2	:	.9550965
PSEUDO-(L)-R2	:	.9550965
ERROR VARIANCE	:	.2333149E-02
LOG-LIKELIHOOD	:	.2674807E+03

E(Y)
-6
-4
-2
0
2
4
6
.93851
1.37939
1.82027
2.26115
2.70202
3.14290
STANDARDIZED RESIDUAL

ANALYSIS OF RESIDUALS

AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS

N= 166

DIFFERENCE	LAGS	AUTOCORRELATIONS											STD-ERROR
													ROW
$(1-B)^0(1-B^{12})^0$	1-12	.30	.10	.13	.09	.03	.03	.05	-.01	.00	-.05	.05	.31	.08
VAR = .2333E-02	13-24	-.05	-.22	-.12	-.08	-.11	-.10	-.07	-.15	-.14	-.21	-.09	.07	.09
	25-36	-.07	-.29	-.11	-.04	-.06	-.06	-.04	-.09	-.09	-.05	.04	.18	.11

DIFFERENCE	LAGS	PARTIAL AUTOCORRELATIONS											STD-ERROR
													ROW
$(1-B)^0(1-B^{12})^0$	1-12	.30	.01	.11	.02	-.01	.01	.03	-.04	.01	-.07	.09	.31	.08
	13-24	-.27	-.21	-.06	-.00	-.01	-.04	-.05	-.09	-.05	-.14	.00	.02	.08
	25-36	.04	-.19	.01	-.04	.01	-.05	-.07	-.05	-.03	.05	-.02	.05	.08

```
INITIALIZATION CARDS FOR CONTROL ROUTINE FOLLOW - - ID CARD - NO. PARAMETERS , NO. STEPS , STEP SIZE ,
MINIMIZATION TOLERANCE - STARTING VALUES OF PARAMETERS - RELATIVE UNCERTAINTIES OF PARAMETERS (ERROR ESTIMATES) -
AND STARTING DIRECTION (RELATIVE PARAMETER INCREMENTS FOR 1ST STEP)

AUTOCORRELATION(1,12) - 1 LAMBDA(X) - 1 LAMBDA(Y) - 1 DELTA - 1 LAMBDA(E)
        6        500     .1E+01     .1E-06
 0.              0.              .10000E+01    .10000E+01   0.             .10000E+01
  .10000E+01     .10000E+01      .10000E+01    .10000E+01    .10000E+01    .10000E+01
 0.              0.              0.            0.            0.            0.

FIRST ENTRY TO FCN , PARAMETERS OF CURRENT STEP FOLLOW - - F,X(I)
  -.2674807096E+03  0.              0.              .10000E+01    .10000E+01   0.             .10000E+01

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY    6  FOLLOW - - F,X(I)
  -.2963635648E+03    .42216E+00    .46449E+00    .98870E+00    .93471E+00    .33239E-01    .10000E+01

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   10  FOLLOW - - F,X(I)
  -.3032005974E+03    .13039E+00    .93696E+00    .88847E+00    .10601E+01    .23587E+00    .10055E+01

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   13  FOLLOW - - F,X(I)
  -.3035295151E+03    .90147E-01    .97559E+00    .88915E+00    .98448E+00    .24080E+00    .10032E+01

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   17  FOLLOW - - F,X(I)
  -.3041157533E+03    .58267E-01    .11213E+01    .96246E+00    .95322E+00    .91033E-01    .85882E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   20  FOLLOW - - F,X(I)
  -.3046195280E+03    .85275E-01    .11715E+01    .96640E+00    .89291E+00    .19108E+00    .76245E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   22  FOLLOW - - F,X(I)
  -.3048122864E+03    .66582E-01    .12463E+01    .11364E+01    .90043E+00    .28379E+00    .77137E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   26  FOLLOW - - F,X(I)
  -.3053677760E+03    .17150E-01    .14839E+01    .86064E+00    .90646E+00    .33573E+00    .84508E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   28  FOLLOW - - F,X(I)
  -.3054463594E+03    .24140E-01    .15154E+01    .79231E+00    .91149E+00    .32728E+00    .69426E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   30  FOLLOW - - F,X(I)
  -.3054718220E+03    .15996E-01    .15679E+01    .78895E+00    .90225E+00    .33489E+00    .59009E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   32  FOLLOW - - F,X(I)
  -.3054782597E+03    .17967E-01    .15828E+01    .80508E+00    .89990E+00    .34959E+00    .58463E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   34  FOLLOW - - F,X(I)
  -.3054784924E+03    .17360E-01    .15881E+01    .79974E+00    .90027E+00    .35270E+00    .57864E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   36  FOLLOW - - F,X(I)
  -.3054786392E+03    .17512E-01    .15877E+01    .80090E+00    .90079E+00    .36056E+00    .54583E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   38  FOLLOW - - F,X(I)
  -.3054786960E+03    .17555E-01    .15863E+01    .80036E+00    .90023E+00    .36429E+00    .53750E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   40  FOLLOW - - F,X(I)
  -.3054786988E+03    .17537E-01    .15861E+01    .80083E+00    .90038E+00    .36467E+00    .53767E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   41  FOLLOW - - F,X(I)
  -.3054786988E+03    .17537E-01    .15861E+01    .80084E+00    .90037E+00    .36470E+00    .53759E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   42  FOLLOW - - F,X(I)
  -.3054786988E+03    .17537E-01    .15861E+01    .80084E+00    .90037E+00    .36470E+00    .53761E+00

NEW ABSOLUTE MINIMUM , PARAMETERS FOR FCN ENTRY   44  FOLLOW - - F,X(I)
  -.3054786988E+03    .17537E-01    .15861E+01    .80084E+00    .90037E+00    .36470E+00    .53761E+00
```

```
THE FUNCTION (FCN) HAS NOT IMPROVED BY     .10000E-06 AFTER 3 STEPS WHICH INCLUDE A MINIMUM - - -

THEREFORE THE RUN HAS BEEN TERMINATED
  -.3054786988E+03    .17537E-01    .15861E+01    .80084E+00    .90037E+00    .36470E+00    .53761E+00

ALL NUMBERS OF LAST ENTRY REFER TO FINAL PT. OF RUN

THE LOWEST VALUE OF THE FUNCTION (FCN) WAS  -.3054786988E+03 FOR ENTRY    44 - - X(I) FOLLOWS
  .17537E-01    .15861E+01    .80084E+00    .90037E+00    .36470E+00    .53761E+00


RUN STATISTICS FOLLOW - -

NFCN=    44 NST=    17

MEAN PROBABILITY OF V TO BE AT THE LIMIT = 0.0000

MEAN (V)           :    2.0233914

E(V) AT THE MEANS  :    1.9998385

MEAN (V**)         :     .0318155

PSEUDO-(E)-R2      :     .9714371

PSEUDO-(L)-R2      :     .9715909

ERROR VARIANCE     :    .9294425E-03

LOG-LIKELIHOOD     :    .3054787E+03
```

E(Y)
-6
-4
-2
0
2
4
6
.92982
1.37685
1.82388
2.27091
2.71794
3.16497
STANDARDIZED RESIDUAL

ANALYSIS OF RESIDUALS

AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS

N= 166

DIFFERENCE	LAGS	AUTOCORRELATIONS												STD-ERROR ROW
$(1-B)^0 (1-B^{12})^0$	1-12	.12	-.01	.03	-.00	-.09	-.04	.02	.05	.09	-.01	.04	-.16	.08
VAR = .9294E-03	13-24	-.15	.05	-.04	-.02	-.01	-.05	.04	-.07	-.09	-.05	.05	-.12	.08
	25-36	-.06	-.16	-.02	.04	-.00	.05	.02	-.06	-.03	-.08	-.14	-.02	.09

DIFFERENCE	LAGS	PARTIAL AUTOCORRELATIONS												STD-ERROR ROW
$(1-B)^0 (1-B^{12})^0$	1-12	.12	-.03	.04	-.01	-.09	-.02	.03	.05	.09	-.04	.04	-.18	.08
	13-24	-.10	.09	-.05	.01	-.05	-.09	.07	-.09	-.02	-.04	.05	-.14	.08
	25-36	-.08	-.17	.03	.03	.01	.01	-.03	-.10	-.03	-.11	-.08	-.04	.08

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS									
			TLK	RFK	SFK	CSFMR	RCUTP	ANT	ANTL1	STR	EX7	EX8
1	.0044	40.61	.0904	.0045	.0696	.0000	.0011	.0041	.0006	.0158	.0127	.0013
2	.0135	23.30	.0032	.0046	.0249	.0297	.0206	.0035	.0017	.0002	.0000	.0201
3	.0171	20.73	.0006	.0002	.0089	.1118	.0077	.0023	.0009	.0116	.0045	.0150
4	.0302	15.58	.0147	.0021	.0001	.0036	.0003	.0085	.0374	.0006	.1073	.0021
5	.0400	13.54	.0037	.0066	.0022	.0143	.1070	.0000	.0113	.0004	.5194	.5597
6	.0606	11.00	.0256	.0106	.0019	.0025	.0642	.0065	.0086	.0141	.0070	.0883
7	.1044	8.38	.0735	.0206	.0006	.0049	.0005	.0071	.0000	.0480	.0000	.0139
8	.2457	5.46	.0060	.0001	.0303	.0783	.0402	.0008	.0521	.0000	.1636	.1159
9	.3019	4.93	.0178	.0098	.0189	.0478	.1045	.0428	.0051	.0041	.0029	.0124
10	.3616	4.50	.0170	.0026	.4818	.4474	.0402	.0009	.0488	.0060	.0196	.0323
11	.4130	4.21	.0098	.0220	.0679	.0942	.2534	.0131	.0397	.0002	.0019	.0000
12	.4841	3.89	.0011	.0206	.0001	.0004	.2623	.0246	.0150	.0094	.0407	.0000
13	.5448	3.67	.0064	.0086	.0732	.0005	.0023	.0186	.0494	.0224	.0180	.0089
14	.6591	3.33	.3523	.3925	.0070	.0091	.0003	.0249	.0782	.0277	.0018	.0005
15	.6825	3.28	.0746	.0203	.0632	.0043	.0057	.1848	.0884	.0334	.0152	.0122
16	.7195	3.19	.0846	.0146	.0006	.0121	.0314	.1562	.0233	.0062	.0014	.0396
17	.8471	2.94	.0100	.0290	.0013	.0102	.0173	.0721	.0301	.4092	.0252	.0121
18	.9600	2.76	.0009	.0579	.0000	.0023	.0015	.2275	.0384	.2015	.0009	.0001
19	1.0349	2.66	.0302	.1743	.0067	.0101	.0134	.0675	.0405	.0357	.0010	.0097
20	1.1215	2.56	.0072	.0229	.0077	.0039	.0009	.0265	.4023	.0433	.0036	.0013
21	1.1920	2.48	.1062	.0848	.0064	.0117	.0176	.0329	.0051	.0694	.0123	.0021
22	1.3289	2.35	.0177	.0519	.0317	.0215	.0004	.0135	.0006	.0000	.0001	.0165
23	1.6109	2.13	.0153	.0034	.0351	.0165	.0030	.0247	.0058	.0062	.0021	.0092
24	1.7766	2.03	.0075	.0027	.0114	.0140	.0001	.0009	.0062	.0142	.0112	.0005
25	1.8728	1.98	.0229	.0244	.0367	.0357	.0000	.0158	.0009	.0087	.0149	.0127
26	2.0494	1.89	.0000	.0032	.0038	.0084	.0000	.0084	.0094	.0105	.0027	.0029
27	2.1939	1.83	.0002	.0052	.0076	.0042	.0002	.0118	.0001	.0010	.0098	.0106
28	7.3297	1.00	.0008	.0000	.0004	.0005	.0038	.0000	.0001	.0001	.0001	.0000

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS									
			EX9	EX0	EX1	DUMZA	FAD	NC	P	RAWEM	TW	TT
1	.0044	40.61	.0009	.0059	.0216	.0422	.0656	.0026	.1382	.5546	.0022	.0017
2	.0135	23.30	.0031	.0055	.0039	.0032	.0646	.4452	.0771	.1507	.0050	.0033
3	.0171	20.73	.0004	.0075	.0317	.0722	.0588	.5095	.3750	.1273	.0003	.0013
4	.0302	15.58	.0001	.0000	.0055	.0663	.3475	.0150	.1558	.1615	.2285	.0035
5	.0400	13.54	.6481	.5474	.4756	.0049	.0033	.0164	.1365	.0035	.2289	.0075
6	.0606	11.00	.2157	.3262	.2311	.0073	.0441	.0073	.0416	.0007	.4897	.0029
7	.1044	8.38	.0016	.0275	.0571	.0824	.3957	.0012	.0715	.0007	.0268	.0681
8	.2457	5.46	.0076	.0179	.0638	.0012	.0013	.0010	.0003	.0000	.0109	.6000
9	.3019	4.93	.0019	.0027	.0048	.3569	.0098	.0005	.0000	.0006	.0022	.0145
10	.3616	4.50	.0010	.0071	.0147	.0328	.0008	.0000	.0001	.0000	.0002	.0001
11	.4130	4.21	.0009	.0023	.0017	.2478	.0001	.0000	.0008	.0000	.0010	.0086
12	.4841	3.89	.0177	.0051	.0000	.0103	.0001	.0001	.0000	.0000	.0011	.0011
13	.5448	3.67	.0079	.0001	.0078	.0071	.0005	.0001	.0001	.0000	.0000	.0108
14	.6591	3.33	.0132	.0000	.0062	.0043	.0010	.0000	.0001	.0000	.0000	.0191
15	.6825	3.28	.0005	.0004	.0031	.0028	.0010	.0001	.0004	.0000	.0000	.0770
16	.7195	3.19	.0072	.0062	.0059	.0014	.0001	.0001	.0003	.0000	.0003	.0949
17	.8471	2.94	.0098	.0007	.0103	.0000	.0008	.0000	.0000	.0000	.0000	.0074
18	.9600	2.76	.0008	.0000	.0001	.0031	.0000	.0000	.0001	.0000	.0002	.0024
19	1.0349	2.66	.0026	.0058	.0044	.0002	.0011	.0000	.0000	.0000	.0000	.0002
20	1.1215	2.56	.0008	.0000	.0008	.0015	.0000	.0000	.0000	.0000	.0001	.0098
21	1.1920	2.48	.0201	.0000	.0064	.0008	.0004	.0000	.0000	.0000	.0000	.0005
22	1.3289	2.35	.0150	.0008	.0133	.0037	.0012	.0000	.0000	.0000	.0004	.0152
23	1.6109	2.13	.0036	.0003	.0032	.0031	.0000	.0000	.0000	.0000	.0004	.0046
24	1.7766	2.03	.0090	.0219	.0151	.0007	.0002	.0001	.0002	.0000	.0000	.0250
25	1.8728	1.98	.0018	.0000	.0003	.0002	.0001	.0000	.0000	.0000	.0003	.0009
26	2.0494	1.89	.0036	.0059	.0101	.0234	.0000	.0001	.0002	.0001	.0000	.0113
27	2.1939	1.83	.0053	.0028	.0013	.0190	.0008	.0000	.0001	.0000	.0003	.0068
28	7.3297	1.00	.0000	.0000	.0001	.0012	.0011	.0003	.0006	.0002	.0010	.0017

NO.	EIGENVALUE	CONDITION INDEX	VARIANCE DECOMPOSITION PROPORTIONS							
			T2T	EANFP	SHONF	WD	SAT	SHD	ZA	CONSTANT
1	.0044	40.61	.3287	.0186	.1949	.3225	.2825	.3265	.0134	.9704
2	.0135	23.30	.3083	.0000	.2840	.4778	.4265	.4273	.0053	.0026
3	.0171	20.73	.0101	.0001	.0019	.1488	.1405	.1600	.0288	.0025
4	.0302	15.58	.0175	.0299	.0005	.0397	.0538	.0555	.0245	.0235
5	.0400	13.54	.0131	.1122	.0131	.0003	.0006	.0014	.0154	.0003
6	.0606	11.00	.0803	.0519	.0001	.0016	.0004	.0000	.0105	.0000
7	.1044	8.38	.0508	.0556	.0290	.0007	.0003	.0006	.0017	.0001
8	.2457	5.46	.0501	.0040	.0009	.0001	.0004	.0000	.0008	.0000
9	.3019	4.93	.0269	.4766	.0072	.0000	.0024	.0003	.0240	.0002
10	.3616	4.50	.0004	.0267	.0224	.0003	.0067	.0010	.0103	.0000
11	.4130	4.21	.0009	.0001	.0036	.0001	.0001	.0000	.3096	.0000
12	.4841	3.89	.0194	.0859	.0115	.0003	.0032	.0000	.2987	.0000
13	.5448	3.67	.0023	.0007	.3124	.0001	.0010	.0001	.1676	.0000
14	.6591	3.33	.0004	.0077	.0038	.0000	.0084	.0005	.0001	.0000
15	.6825	3.28	.0000	.0091	.0318	.0001	.0105	.0096	.0001	.0001
16	.7195	3.19	.0093	.0026	.0255	.0009	.0094	.0035	.0003	.0001
17	.8471	2.94	.0025	.0274	.0002	.0001	.0052	.0004	.0002	.0000
18	.9600	2.76	.0009	.0002	.0169	.0002	.0234	.0020	.0124	.0000
19	1.0349	2.66	.0328	.0157	.0073	.0000	.0007	.0016	.0042	.0000
20	1.1215	2.56	.0004	.0024	.0000	.0005	.0151	.0003	.0021	.0000
21	1.1920	2.48	.0081	.0055	.0121	.0004	.0000	.0006	.0065	.0000
22	1.3269	2.35	.0237	.0008	.0001	.0010	.0026	.0001	.0011	.0001
23	1.6109	2.13	.0068	.0059	.0009	.0024	.0047	.0047	.0017	.0000
24	1.7766	2.03	.0010	.0004	.0033	.0000	.0001	.0002	.0018	.0000
25	1.8728	1.98	.0000	.0004	.0000	.0016	.0010	.0033	.0011	.0000
26	2.0494	1.89	.0028	.0332	.0001	.0000	.0001	.0002	.0335	.0000
27	2.1939	1.83	.0017	.0264	.0154	.0001	.0000	.0002	.0231	.0000
28	7.3297	1.00	.0001	.0000	.0013	.0001	.0003	.0001	.0013	.0001

CORRELATION MATRIX

TLK	1.0000														
RFK	-.2416	1.0000													
SFK	.1958	-.0955	1.0000												
CSFMR	.1151	-.1012	.5950	1.0000											
RCUTP	.0464	-.0624	.0665	.0966	1.0000										
ANT	.0617	-.1199	-.0349	-.0216	.0408	1.0000									
ANTL1	.0511	.0701	-.0197	-.0066	-.0459	-.0095	1.0000								
STR	-.0232	-.0365	-.0011	-.0012	-.1025	-.0000	-.0000	1.0000							
EX7	.0762	-.0408	.0045	-.0016	-.2802	-.0000	-.0000	.2092	1.0000						
EX8	-.0957	-.0683	-.0034	-.0014	.0646	-.0000	-.0000	-.1045	-.4933	1.0000					
EX9	.0409	.0942	-.0006	-.0010	.0102	-.0000	-.0000	-.0000	-.0001	-.4644	1.0000				
EX0	.0694	-.0703	-.0008	-.0010	-.0195	-.0000	-.0000	-.0000	-.0001	-.0001	-.4973	1.0000			
EX1	-.1126	.0320	-.0147	-.0212	-.0400	-.0005	-.0005	-.0008	-.0012	-.0012	-.0009	-.6845	1.0000		
DUMZA	-.0560	-.0624	-.0324	.0850	.1064	-.2134	-.2047	-.1224	-.0174	.0132	.0024	.0013	-.0240	1.0000	
FAD	.1492	.0677	-.1296	-.1386	-.1533	-.0757	.1005	.2000	.4278	-.3011	.1263	-.0165	-.0109	.0979	1.0000
NC	.0378	-.0201	-.0131	-.0728	.3490	-.0072	-.0561	-.0273	-.1511	.2029	.0279	.0214	.0565	-.0035	-.2022
P	.1089	-.0333	.0477	.0091	.3049	-.0496	-.0775	.0044	.0018	.0205	.1391	.0061	.0477	.1747	-.0000
RAWEM	.0498	-.0624	.1617	.2270	.3940	.0485	-.0468	-.1227	-.2353	.1591	-.0834	.0127	.1870	.0095	-.4436
TW	.0276	.0240	.1172	.1069	.2456	-.0866	-.1249	-.0729	-.5419	.2239	-.0079	.1396	-.1005	-.0180	-.5357
TT	-.0193	-.0092	.1491	.2357	.2024	-.0366	-.2004	-.1400	-.4320	.0365	-.0116	.0069	.0012	.1154	-.4508
T2T	.0604	-.0221	-.0867	-.0654	.2897	-.0202	-.0210	-.0280	-.0107	-.0115	-.0075	-.0058	.2120	.1334	.2589
EANFP	.0291	-.0251	-.0750	-.0126	.0382	-.0655	-.0784	-.0513	.0288	-.0285	.0827	-.0376	-.1642	.5558	.1908
SHONF	-.1180	.0473	-.1464	-.1261	-.2356	-.0585	.0410	.0393	.1998	-.1764	-.0408	-.0275	.0621	.1698	.6608
WD	-.0161	-.0971	.1007	.1501	.1088	.1729	-.0619	-.0258	-.0190	.0337	-.0150	.0240	-.0082	-.0391	-.1896
SAT	.0396	.0405	.0472	.0624	.0190	.0070	-.0952	-.0073	-.0308	.0472	-.0293	.0084	-.0109	-.0202	-.0440
SHD	.0198	.0757	-.0513	-.1929	-.0520	-.1914	.1243	.0179	-.0516	.0086	.0600	-.0210	-.0094	-.0196	-.0565
ZA	-.0394	.0647	.0334	.1192	-.0372	-.2055	-.0384	-.1129	-.0029	-.0029	-.0023	-.0024	-.0487	.4606	.0671
XCANFA	.0406	-.0677	.0199	.0153	-.1834	.0228	.0386	-.5866	.5925	-.1478	-.0403	-.0304	.0084	.0430	.1065
	TLK	RFK	SFK	CSFMR	RCUTP	ANT	ANTL1	STR	EX7	EX8	EX9	EX0	EX1	DUMZA	FAD

CORRELATION MATRIX

NC	1.0000												
P	.7670	1.0000											
RAWEM	.6825	.5416	1.0000										
TW	.3187	.3012	.4641	1.0000									
TT	-.1380	-.1621	.1223	.3947	1.0000								
T2T	.6449	.5134	.5107	-.0826	-.2763	1.0000							
EANFP	-.1878	-.1056	-.2574	-.3475	-.0126	-.0474	1.0000						
SHONF	-.5183	-.4826	-.6776	-.6357	-.2042	.0590	.2828	1.0000					
WD	.1594	.0928	.2281	-.0283	.0524	-.0223	.0128	-.2570	1.0000				
SAT	.0566	.0623	.0668	.2079	.0394	.0188	-.1328	-.0723	-.4928	1.0000			
SHD	.0384	.0802	-.0062	.2254	.0158	-.0145	-.0775	-.1180	-.7595	.0124	1.0000		
ZA	-.1446	-.1308	-.0824	-.0598	.1188	-.0436	.4490	.1680	-.1097	.0614	.0558	1.0000	
XCANFA	-.0544	-.0141	-.0341	-.4068	-.3900	.0293	.1077	.0850	.1333	-.0656	-.1551	.0954	1.0000
	NC	P	RAWEM	TW	TT	T2T	EANFP	SHONF	WD	SAT	SHD	ZA	XCANFA

EQUATION : ACAMPA

VARIABLE	LAMBDA(X) TYPE	LAMBDA(X) GROUP	COEFFICIENT	GRADIENT	STD-ERROR	STUDENT-T	ELAST(S) (OBS.NO.	ELAST(E) 13- 178)	ELAST(S) (OBS.NO.	ELAST(E) 167- 178)
1 TLR	NL	0	-.27723430E-02	.81378540E-09	.75510960E-02 .73219650E-02	-.3671 -.3786	-.0020	-.0029	-.0031	-.0031
2 RPA	NL	0	-.21784360E-01	-.20691020E-10	.78365640E-02 .71761910E-02	-2.7811 -3.0370	-.0083	-.0084	-.0074	-.0074
3 SPA	NL	0	.29084630E-04	.66116810E-09	.12085780E-02 .11383690E-02	.0241 .0255	.0000	.0000	.0001	.0001
4 CSTAR	NL	0	.16538190E-02	.30320170E-09	.26583660E-02 .24922410E-02	.6146 .6556	.0022	.0022	.0032	.0032
5 RCLUS	NL	0	-.10237480E-01	.20374240E-09	.11009950E-01 .10733630E-01	-1.6565 -1.6991	-.0221	-.0223	-.0439	-.0441
6 RAT	NL	0	-.58712340E-01	.56274980E-10	.28625400E-01 .27426370E-01	-2.4004 -2.5053	-.0002	-.0002	0.0000	0.0000
7 ANTLI	NL	0	.34065320E-01	-.21827870E-10	.19192940E-01 .18135840E-01	1.2539 1.3269	.0001	.0001	0.0000	0.0000
8 LOD	NL	0	-.59912080E+00	-.81781930E-09	.49913210E-01 .13996710E-01	-14.1909 -49.5917	-.0099	-.0099	0.0000	0.0000
9 PAT	NL	0	.26116630E+00	.63790910E-09	.30221110E-01 .78454540E-02	8.3110 32.0142	.0149	.0148	0.0000	0.0000
10 SAD	NL	0	.23272180E-01	.40472010E-10	.54009890E-02 .43304920E-02	4.3089 5.3740	.0031	.0031	0.0000	0.0000
11 EAD	NL	0	.19010490E-01	.14210850E-10	.74291400E-02 .87711570E-02	2.0205 2.2168	.0011	.0012	0.0000	0.0000
12 DAD	NL	0	.22738910E-02	.22077080E-09	.10686550E-01 .10359140E-01	.2125 .2195	.0001	.0002	0.0000	0.0000
13 DAI	NL	0	.78123670E-03	.42075500E-09	.15957300E-01 .15659050E-01	.0490 .0499	.0000	.0000	.0007	.0007
14 DUMDO	NL	0	-.82447940E-01	.51212720E-09	.21117330E-01 .18489790E-01	-3.9043 -4.4591	-.0184	-.0186	-.0463	-.0465
15 FAD	EX	1	-.19038710E+01	-.26090770E-09	.13034800E+01 .21941470E+00	-1.1536 -6.8531	-.1886	-.1906	-.2542	-.2550
16 MC	EX	1	-.28073270E+00	.19500630E-08	.20140700E+00 .09714120E-01	-1.4230 -3.1961	-.3772	-.3812	-.5449	-.5467
17 P	EX	1	.86161390E-01	.28472100E-09	.40370880E+00 .35314380E+00	.2134 .2440	.0599	.0605	.0714	.0716
18 RHOPW	EX	1	.19185030E+00	.27003470E-08	.19830380E+00 .63758060E-01	.9675 3.0091	.5339	.5396	.6735	.6757
19 TW	EX	1	-.59420500E-02	.14118340E-08	.42200380E-01 .40760540E-01	-.1408 -.1458	-.0096	-.0097	-.0120	-.0120
20 TI	EX	1	-.59729030E+00	.12489160E-08	.20500710E+00 .35811570E-01	-2.4733 -14.1647	-.4953	-.5005	-.4782	-.4798

21 T2T	EL	1	.23171998E+00	.42341248E-10	.22284038E+00 .22207528E+00	1.0408 1.0434	.1221	.1234	.1460	.1465
22 EANTP	EL	1	.55436018E+00	.93648538E-11	.15208708E+00 .12560098E+00	3.6450 4.4137	.3481	.3518	.3523	.3534
23 DHDMP	EL	1	.61291098E-01	.38059518E-09	.59471898E-01 .51938298E-01	1.0306 1.1801	.0582	.0556	.0573	.0564
24 WD	EL	1	.77883918E+00	.51567288E-09	.33832438E+00 .16128638E+00	2.2951 4.8171	.7411	.7490	.7858	.7884
25 SAT	EL	1	.63401598E-01	.10831028E-08	.50989108E-01 .20468778E-01	1.2434 3.0982	.1090	.1101	.1153	.1157
26 SHD	EL	1	.49817048E-01	-.11235108E-09	.46180588E-01 .21230618E-01	1.0787 2.3465	.0999	.1009	.1056	.1060
27 ZA	EL	1	.21898138E+00	.68058748E-10	.11905888E+00 .11468438E+00	1.8393 1.9099	.0640	.0646	0.0000	0.0000
28 CONSTANT	NL	0	-.15322698E+01	.33147888E-09	.92137228E+00 .69316318E+00	-1.6630 -2.2105	-.8124	-.8210	-.8611	-.8639
AUTOCORRELATION										
29 RHO 1			.17535608E-01	.17529548E-04	.48001658E-01 .46482058E-01	.3653 .3773				
30 RHO 12			.91955118E+00	.29743058E-04	.43723528E-01 .43295248E-01	21.0310 21.2391				
BOX-COX ON (X)										
31 LAMBDA(X) 1			.80083608E+00	.22157808E-05	.47927468E+00	1.6709 -.4156				
BOX-COX ON (Y)										
32 LAMBDA(Y)			.90037438E+00	-.51847878E-05	.13046398E+00	6.9013 -.7636				
HETEROSKEDASTICITY										
33 DELTA 1			.38489778E+00	.19644518E-05	.11636388E+01 .34390668E+00	.3134 1.0605				
34 LAMBDA(Z) 1			.53760668E+00	.20223638E-06	.39406028E+01	.1364 -.1173				
RESIDUAL (M)										
35 ERROR VARIANCE			.92944258E-03	.20387238E-07	.53922218E-03 .31921638E-03	1.7237 2.9116				

DERIVATIVES OF Y AND E(V) WITH RESPECT TO THE INDEPENDENT VARIABLES (ORIGINAL UNITS)

	VARIABLE	DERIVATIVE(S) (OBS.NO.	DERIVATIVE(E) 13- 178)	DERIVATIVE(S) (OBS.NO.	DERIVATIVE(E) 167- 178)
1	TLR	-.2973998E+03	-.2970478E+03	-.2954895E+03	-.2953749E+03
2	RPA	-.2337962E+06	-.2335196E+06	-.2322946E+06	-.2322045E+06
3	SPA	.3120017E+03	.3116326E+03	.3099977E+03	.3098775E+03
4	CSFMA	.1752659E+04	.1750585E+04	.1741402E+04	.1740726E+04
5	RCUTP	-.1956400E+02	-.1954086E+02	-.1943834E+02	-.1943081E+02
6	ANT	-.7371030E+06	-.7362310E+06	-.7323687E+06	-.7320846E+06
7	ANTLI	.2581577E+06	.2578523E+06	.2564996E+06	.2564001E+06
8	STR	-.7446093E+06	-.7437285E+06	-.7398268E+06	-.7395399E+06
9	EX7	.2694355E+01	.2691167E+01	.2677049E+01	.2676011E+01
10	EX8	.2496494E+01	.2493541E+01	.2480459E+01	.2479497E+01
11	EX9	.1610230E+01	.1608325E+01	.1599888E+01	.1599268E+01
12	EXO	.2439287E+00	.2436402E+00	.2423620E+00	.2422680E+00
13	EX1	.8380618E-01	.8370704E-01	.8326790E-01	.8323561E-01
14	DUMZA	-.8844499E+06	-.8834036E+06	-.8787692E+06	-.8784284E+06
15	FAD	-.2308566E+08	-.2305835E+08	-.2160705E+08	-.2159867E+08
16	HC	-.2453730E+02	-.2450827E+02	-.2257243E+02	-.2256367E+02
17	P	.8641424E+04	.8631201E+04	.8339018E+04	.8335784E+04
18	RAWEM	.1362609E+06	.1360997E+06	.1296517E+06	.1296015E+06
19	TW	-.4835612E+05	-.4829891E+05	-.4611091E+05	-.4609303E+05
20	TT	-.4674813E+06	-.4669283E+06	-.4753858E+06	-.4752014E+06
21	T2T	.2489301E+06	.2486356E+06	.2400395E+06	.2399464E+06
22	EANFP	.5701815E-01	.5695069E-01	.5730829E-01	.5728607E-01
23	SHONF	.5594331E+01	.5283919E+01	.4419871E+01	.4334739E+01
24	WG	.7200665E+06	.7192146E+06	.7153866E+06	.7151092E+06
25	SAT	.5076464E+06	.5070459E+06	.5046184E+06	.5044227E+06
26	SHD	.3838785E+06	.3834243E+06	.3816305E+06	.3814825E+06
27	ZA	.2724442E+08	.2721219E+08	0.	0.

LOG-LIKELIHOOD

- INITIAL VALUE : 267.4807096
- FINAL VALUE : 305.4786988

- RATIO TEST : [illegible]

PSEUDO-(E)-R2 : .9714371

PSEUDO-(L)-R2 : .9715909

STD-ERROR OF (W) : .0304868

NUMBER OF OBSERVATIONS		FIRST-LAST OBS.NO.
- SAMPLE	: 181	1 - 181
- ESTIMATION	: 166	13 - 178
- PREDICTION	: 3	179 - 181

PREDICTION OF THE DEPENDENT VARIABLE FOR 3 PERIODS (ORIGINAL UNITS)

	OBSERVED VALUE	SIMULATED VALUE	MAX. LIKELIHOOD PREDICTED VALUE	ERROR OF FORECAST IN %	STD-ERROR OF FORECAST	95% CONFIDENCE INTERVAL LOWER BOUND	UPPER BOUND
179	.1923298E+08	.1937022E+08	.1936936E+08	-.71	.4810096E+06	.1842658E+08	.2031213E+08
180	.2042556E+08	.1976263E+08	.1976167E+08	3.25	.5171737E+06	.1874800E+08	.2077533E+08
181	.1954108E+08	.2010612E+08	.2010487E+08	-2.89	.6350315E+06	.1886021E+08	.2134954E+08

BIBLIOGRAPHY

Belsley, D., E. Kuh, and R.E. Welsh (1980), Regression Diagnostics, John Wiley, New York.

Berndt, E.K., B.H. Hall, R.E. Hall and J.A. Hausman (1974), "Estimation and Inference in Nonlinear Structural Models", Annals of Economic and Social Measurement, pp. 653-665.

Box, G.E.P. and G.M. Jenkins (1976), Time Series Analysis : Forecasting and Control, Holden-Day, San Francisco. Revised Edition.

Dagenais, M.G., M.J.I. Gaudry and T.C. Liem (1980), "Multiple Regression Analysis with Box-Cox Transformation and Nonspherical Residual Errors : A Transportation Application", Publication #166, Centre de recherche sur les transports, Université de Montréal.

Dagenais, M.G., M.J.I. Gaudry and T.C. Liem (1985), "Urban Travel Demand : The Impact of Box-Cox Transformations with Nonspherical Residual Errors", Publication #358, Centre de recherche sur les transports, Université de Montréal.

Fletcher, R. and M.J.D. Powell (1963), "A Rapidly Convergent Descent Method for Minimization", Computer Journal, vol. 6, pp. 163-168.

Gaudry, M.J.I. and M.G. Dagenais (1979), "The Use of Box-Cox Transformations in Regression Models with Heteroskedastic Autoregressive Residuals", Cahier No. 7814, Department of Economics, Université de Montréal.

Gaudry, M.J.I. and M.G. Dagenais (1979), "Heteroscedasticity and the Use of Box-Cox Transformations", Economics Letters, vol. 2, no. 3, pp. 225-229.

Gaudry, M.J.I. and J.J. Wills (1978), "Estimating the Functional Form of Travel Demand Models", Transportation Research, 12, pp. 253-289.

Judge, G.G., W.E. Griffiths, R.C. Hill, H. Lütkepohl and T.C. Lee (1985), The Theory and Practice of Econometrics, Second Edition, John Wiley, New York.

Kendall, M.G. and A. Stuart (1967), The Advanced Theory of Statistics, Vol. II, Griffin, London.

Tobin, J. (1958), "Estimation of Relationships for Limited Dependent Variables", Econometrica, vol. 25, pp. 24-36.